College Algebra — Joysheet 10
MAT 140, Spring 2021 — D. Ivanšić

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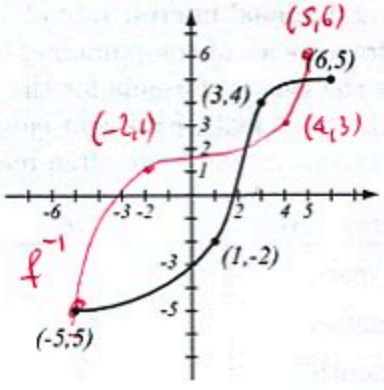
Covers: 5.1–5.3 *Show all your work!*

1. (6pts) The graph of a function f is given.
a) Is this function one-to-one? Justify.
b) If the function is one-to-one, find the graph of f^{-1}, labeling the relevant points.



a) Yes - it passes the horizontal line test

b)

2. (12pts) Let $f(x) = 1 - \dfrac{3}{x-2}$. Find the formula for f^{-1}. Find the ranges of f and f^{-1}.

$y = 1 - \frac{3}{x-2}$

$y - 1 = -\frac{3}{x-2}$

$\frac{1}{y-1} = -\frac{x-2}{3}$ /·(-3)

$\frac{-3}{y-1} = x-2$

$x = 2 - \frac{3}{y-1} = \frac{2(y-1)-3}{y-1} = \frac{2y-5}{y-1} = f^{-1}(y)$

Range f = domain $f^{-1} = (-\infty, 1) \cup (1, \infty)$

Can't have $y-1=0$

$y = 1$

Range f^{-1} = domain $f = (-\infty, 2) \cup (2, \infty)$

Can't have $x-2=0$

$x = 2$

3. (8pts) Evaluate without using the calculator:

$\log_6 36 = 2$ $\quad \log_7 \frac{1}{343} = -3$ $\quad \log_8 32 = \frac{5}{3}$ $\quad \log_{a^2} \sqrt{a} = \frac{1}{4}$

$6^? = 36$ $\quad 7^? = \frac{1}{343} = \frac{1}{7^3} = 7^{-3}$ $\quad 8^? = 32 = 2^5 = (8^{\frac{1}{3}})^5 = 8^{\frac{5}{3}}$ $\quad (a^2)^? = \sqrt{a} = a^{\frac{1}{2}}$ $\quad 2 \cdot ? = \frac{1}{2}$, $? = \frac{1}{4}$

4. (4pts) Use the change-of-base formula and your calculator to find $\log_{27} 4$ with accuracy 6 decimal places. Show how you obtained your number.

$\log_{27} 4 = \frac{\ln 4}{\ln 27} = 0.420620$

5. (12pts) Investigate the effect of increased frequency of compounding: for a deposit of \$10,000 and annual interest rate of 3.75%, calculate the amount in the account after 1 year for the frequencies of compounding below.
a) Write the general formula for the amount, replacing the variables by numbers, if known.
b) Use the table feature on your calculator to quickly compute amounts after 1 year.
c) Does compounding more often make a big difference?

Frequency: every	n	Amount after 1 year
year	1	10,375.00
quarter	4	10,380.31
month	12	10,381.51
day	365	10,382.10
hour	365·24	10,382.12
second	365·24·3600	10,382.12

b)

a) $A = 10000\left(1 + \frac{0.0375}{n}\right)^{n \cdot 1}$

c) After daily compounding, it does not make much difference

6. (3pts) Find the domain of $f(x) = \ln(4 - 6x)$.

Must have $4 - 6x > 0$

$-6x > -4$

$x < \frac{-4}{-6}$

$x < \frac{2}{3}$

$(-\infty, \frac{2}{3})$

7. (8pts) The cost per person of a field trip for x students is given by $C(x) = \dfrac{200 + 32x}{x}$, where C is in dollars.
a) Find the cost per person if 24 or 32 students go.
b) Find a formula for the inverse function and explain what it represents.
c) How many students need to go so that cost per person is \$52? \$37?

a) $C(24) = \$40.33$

$C(32) = \$38.25$

b) $C = \frac{200 + 32x}{x}$

$Cx = 200 + 32x$

$Cx - 32x = 200$

$(C - 32)x = 200$

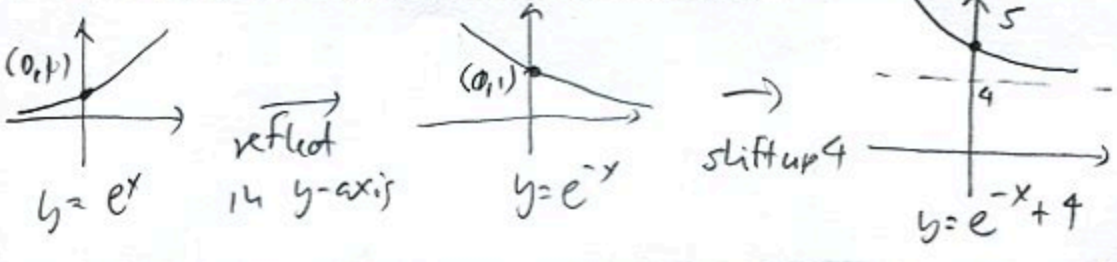
$x = \frac{200}{C - 32}$

$f^{-1}(C) = \frac{200}{C - 32}$ represents the number of students going if C is per-person cost

c) $\frac{200}{52 - 32} = 10$

$\frac{200}{37 - 32} = 40$

8. (7pts) Using transformations, draw the graph of $f(x) = e^{-x} + 4$. Explain how you transform the graph of a basic function in order to get the graph of f. Show at least one point on the graph, and asymptotes to the graph, if any.



$y = e^{x}$ reflect in y-axis $y = e^{-x}$ shift up 4 $y = e^{-x} + 4$