College Algebra — Joysheet 9
MAT 140, Spring 2021 — D. Ivanšić

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Covers: 3.3, 3.4, 3.5 Show all your work!

1. (4pts) Solve the equation.

$|3x-1|=7$

$3x-1=7$ or $3x-1=-7$

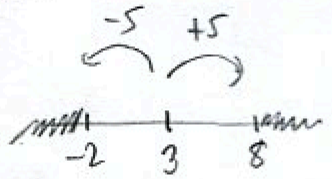
$3x=8$ $\qquad$ $3x=-6$

$x=\frac{8}{3}$ or $x=-2$

2. (12pts) Solve the inequalities. Draw your solution and write it in interval form.

$|x-3|>5$

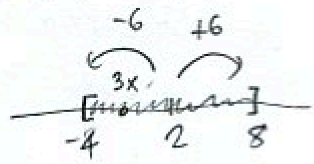
distance from x to $3 > 5$



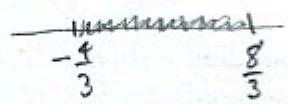
$(-\infty,-2)\cup(8,\infty)$

$|3x-2|\le 6$

distance from $3x$ to $2 \le 6$



$\div 3$



$\left[-\frac{4}{3},\frac{8}{3}\right]$

Solve the equations:

3. (8pts) $\dfrac{x-3}{x+4}+\dfrac{x+1}{x+9}=\dfrac{x^2+4x-5}{x^2+13x+36}$ $\Big|\cdot(x+4)(x+9)$

$(x+4)(x+9)$

$\frac{x-3}{\cancel{x+4}}\cancel{(x+4)}(x+9)+\frac{x+1}{\cancel{x+9}}(x+4)\cancel{(x+9)}=\frac{x^2+4x-5}{\cancel{(x+4)}\cancel{(x+9)}}\cancel{(x+4)}\cancel{(x+9)}$

$(x-3)(x+9)+(x+1)(x+4)=x^2+4x-5$

$x^2+6x-27+x^2+5x+4=x^2+4x-5$

$2x^2+11x-23=x^2+4x-5 \quad \Big|-(x^2+4x-5)$

$x^2+7x-18=0$

$(x+9)(x-2)=0$

$x=-9,2$ $\qquad$ $x=-9$ gives 0 in denom.

$\boxed{x=2 \text{ only solution}}$

4. (8pts) $x=3-\sqrt{33-x}$

$x-3=-\sqrt{33-x} \quad \Big|^2$

$x^2-2\cdot x\cdot 3+3^2=(-1)^2(33-x)$

$x^2-6x+9=33-x \quad \Big|+x-33$

$x^2-5x-24=0$

$(x-8)(x+3)=0$

$x=8,-3$

check: $8\stackrel{?}{=}3-\sqrt{33-8}$

$8\stackrel{?}{=}3-\sqrt{25}$ no

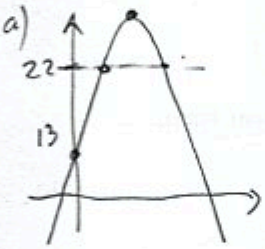
$-3\stackrel{?}{=}3-\sqrt{33+3}$

$-3\stackrel{?}{=}3-6$ yes

$\boxed{x=-3 \text{ only solution}}$

5. (14pts) A marshmallow is launched from height 13 meters upwards with initial velocity 18 meters per second. Its height in meters after t seconds is given by $s(t) = -5t^2 + 18t + 13$.
a) Sketch the graph of the height function.
b) When does the marshmallow reach its greatest height, and what is that height?
c) When is the marshmallow at height 22 meters?

a)



b) $h = -\frac{b}{2a} = -\frac{18}{2(-5)} = \frac{18}{10} = \frac{9}{5} = 1.8$

$k = -5\left(\frac{9}{5}\right)^2 + 18\cdot\frac{9}{5} + 13 = -\cancel{5}\cdot\frac{81}{\cancel{25}_{5}} + 18\cdot\frac{9}{5} + 13 = \frac{-81+162+65}{5} = \frac{146}{5} = 29.2$

After 1.8 s, reaches max height of 29.2 meters

c) $-5t^2+18t+13 = 22$

$-5t^2+18t-9=0$

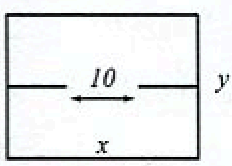
$5t^2-18t+9=0$

$t = \frac{-(-18) \pm \sqrt{(-18)^2-4\cdot5\cdot9}}{2\cdot5} = \frac{18\pm\sqrt{144}}{10}$ $\quad 324-180$

$= \frac{18\pm12}{10} = 3, \frac{3}{5} = 3, 0.6$

It is at height 22 m at times $t = 3, 0.6$ s.

6. (14pts) Cora is building a shed with two rooms and a 10-ft opening between them and has enough money to build 400 feet of walls (see picture). Her goal is to maximize the enclosed area.
a) Express the area of the shed as a function of one of the sides of the rectangle. What is the domain of this function?
c) Sketch the graph of the area function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the shed that has the greatest area and what is the greatest area possible?



a) $A = xy = x\left(205 - \frac{3}{2}x\right) = -\frac{3}{2}x^2 + 205x$

$400 = 2x + x - 10 + 2y = 3x + 2y - 10$

$2y = 410 - 3x$

$y = 205 - \frac{3}{2}x$

Domain:
Must have $x \geq 10$

$y \geq 0$

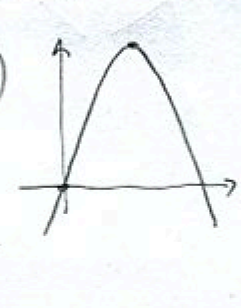
$205 - \frac{3}{2}x \geq 0$

$-\frac{3}{2}x \geq -205 \quad |\cdot\left(-\frac{2}{3}\right)$

$x \leq \frac{410}{3}$

Domain: $\left[10, \frac{410}{3}\right]$

b)



$h = -\frac{b}{2a} = -\frac{205}{\cancel{2}\cdot\left(-\frac{3}{\cancel{2}}\right)} = \frac{205}{3} = 68.333333$

$k = \frac{205}{3}\cdot\left(205 - \frac{\cancel{3}}{2}\cdot\frac{205}{\cancel{3}}\right) = \frac{205}{3}\cdot\frac{205}{2} = \frac{42025}{6}$

$= 7004.166667$

$205 - \frac{3}{2}\cdot 68.333333$

Dimensions: 68.333333×102.5

Max area: 7004.166667 ft^2