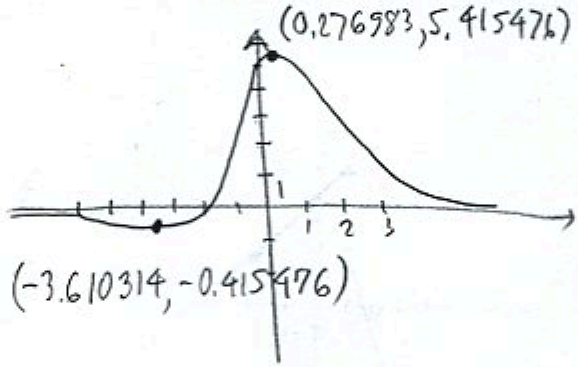
College Algebra — Joysheet 5	**Name:** Saul Ocean
MAT 140, Spring 2021 — D. Ivanšić	*Covers: 2.1, 2.2, 2.3 Show all your work!*

1. (10pts) Use your calculator to accurately sketch the graph of the function $f(x) = \dfrac{3x+5}{x^2+1}$. Draw the graph here, indicate units on the axes, and solve the problems below with accuracy 6 decimal points.

a) Find the local maxima and minima for this function.

b) State the intervals where the function is increasing and where it is decreasing.

a)



$f(0{,}276983) = 5.415476$ is a local max

$f(-3.610314) = -0.415476$ is a local min.

b) Increasing on $(-3.610314,\ 0.276983)$

Decreasing on

$(-\infty, -3.610314)$ and $(0{,}276983, \infty)$

2. (20pts) Let $f(x) = \dfrac{2}{\sqrt{x-3}}$, $g(x) = \dfrac{x-7}{x-5}$. Find the following (simplify where possible):

$(f+g)(4) = f(4) + g(4) = \dfrac{2}{\sqrt{4-3}} + \dfrac{4-7}{4-5}$

$= \dfrac{2}{\sqrt{1}} + \dfrac{-3}{-1} = 2 + 3 = 5$

$(fg)(-2) = f(-2)\cdot g(-2) = \dfrac{2}{\sqrt{-2-3}} \cdot \dfrac{-2-7}{-2-5}$

$\sqrt{-5}$ not defined

not defined

$\dfrac{f}{g}(x) = \dfrac{f(x)}{g(x)} = \dfrac{\frac{2}{\sqrt{x-3}}}{\frac{x-7}{x-5}} = \dfrac{2}{\sqrt{x-3}} \cdot \dfrac{x-5}{x-7}$

$= \dfrac{2(x-5)}{(x-7)\sqrt{x-3}}$

$(f \circ g)(4.5) = f(g(4.5)) = f\left(\dfrac{4.5-7}{4.5-5}\right)$

$= f\left(\dfrac{-2.5}{-0.5}\right) = f(5) = \dfrac{2}{\sqrt{5-3}} = \dfrac{2}{\sqrt{2}} = \sqrt{2}$

$\sqrt{2}\cdot\sqrt{2}$

$(g \circ f)(x) = g(f(x)) = g\left(\dfrac{2}{\sqrt{x-3}}\right) = \dfrac{\frac{2}{\sqrt{x-3}} - 7}{\frac{2}{\sqrt{x-3}} - 5} \cdot \dfrac{\sqrt{x-3}}{\sqrt{x-3}} = \dfrac{2 - 7\sqrt{x-3}}{2 - 5\sqrt{x-3}}$

f

g

3 5 7

The domain of $\dfrac{f}{g}(x)$ in interval notation

Domain of f: must have $x > 3$

Domain of g: can't have $x = 5$

Can't have $g(x) = 0$: $\dfrac{x-7}{x-5} = 0$ $\quad x - 7 = 0$

$x = 7$

Domain of $\frac{f}{g}$ is

$(3,5) \cup (5,7) \cup (7, \infty)$

3. (8pts) Consider the function $h(x) = (x^2 - 4)^3$ and find **two** different solutions to the following problem: find functions f and g so that $h(x) = f(g(x))$, where neither f nor g are the identity function.

$g(x) = x^2 - 4$ $\qquad$ $g(x) = x^2$

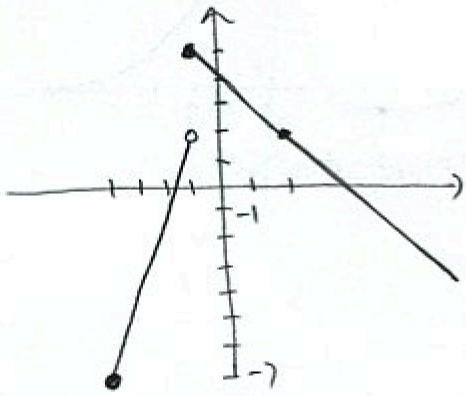
$f(x) = x^3$ $\qquad$ $f(x) = (x-4)^3$

4. (8pts) Sketch the graph of the piecewise-defined function:

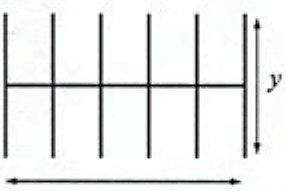
$$f(x) = \begin{cases} 3x + 5, & \text{if } -4 \le x < -1 \\ -x + 4, & \text{if } x \ge -1. \end{cases}$$

x	$3x+5$
-4	-7
-1	2

x	$-x+4$
-1	5
2	2



5. (14pts) A county health department is setting up COVID-19 vaccination and testing bays inside an arena. Each block of ten bays must have total area 2000 square feet. It wishes to minimize construction cost, which is same as minimizing the total length of the walls.
a) Express the total length of the walls of the building as a function of the length of one of the sides x. What is the domain of this function?
b) Graph the function in order to find the minimum. What are the dimensions of the block for which the total length of the walls is minimal? What is the minimal wall length?



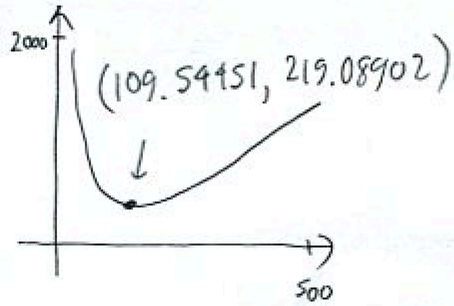
area $= x \cdot y = 2000 \Rightarrow y = \frac{2000}{x}$

a) Total wall length $= \ell = x + 6y = x + 6 \cdot \frac{2000}{x} = x + \frac{12000}{x}$

Domain
must have $x > 0$
No restriction on y,
so no additional
restriction on x

$(0, \infty)$

b)



Dimensions: $x \times y$ $\quad y = \frac{2000}{x}$

$109.54451 \times 18.257419$

Max wall length:

219.08902 ft