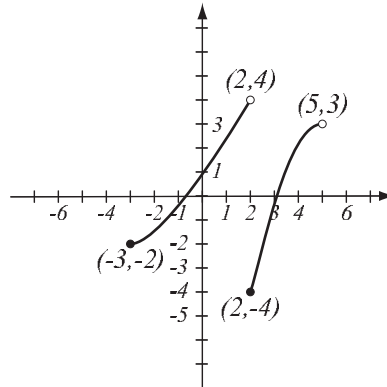


1. (8pts) Use the graph of the function f at right to answer the following questions.

- Find: $f(-3) =$ $f(2) =$
- What is the domain of f ?
- What is the range of f ?
- What are the solutions of the equation $f(x) = 1$?



2. (10pts) Use your calculator to accurately sketch the graph of $y = x^3 - 5x - 9$.

- Draw the graph on paper and indicate units on the axes.
- Find all the x - and y -intercepts (accuracy: 6 decimal points).

3. (5pts) Write the equation of the line that passes through points $(-1, 3)$ and $(7, 1)$.

4. (10pts) Find the equation of the line (in form $y = mx + b$) that is perpendicular to the line $3x + 2y = 8$ and passes through the y -intercept of the given line. Draw both lines.

5. (8pts) Draw the triangle with vertices $A = (-3, -1)$, $B = (6, 4)$ and $C = (1, 7)$. Use either slopes or the Pythagorean theorem to determine if this is a right triangle.

6. (9pts) Let $f(x) = \frac{x^2 + 2x}{\sqrt{x + 3}}$. Find the following (simplify where appropriate).

$$f(1) =$$

$$f(-6) =$$

$$f(3t) =$$

$$f(w - 1) =$$

7. (9pts) Find the domains of the functions below and write them using interval notation.

$$f(x) = \frac{2x - 3}{x^2 - 1}$$

$$g(x) = \sqrt{10 - 4x}$$

8. (7pts) Solve and write the solution in interval notation.

$$3x - 4 < 5 \text{ or } 2x - 7 > 0$$

9. (8pts) A circle has center $(-1, -3)$ and passes through the point $(2, 0)$.

a) Find the equation of the circle.

b) Draw the circle in the coordinate plane.

10. (12pts) Lauren has these options for a data plan for her cell phone:

A) \$12 flat fee for the first GB, and then \$4 per GB for usage beyond the first GB.

B) \$3 flat fee plus \$5.50 per GB.

Assuming Lauren always uses at least 1 GB of data, for which amount of data is plan B better?

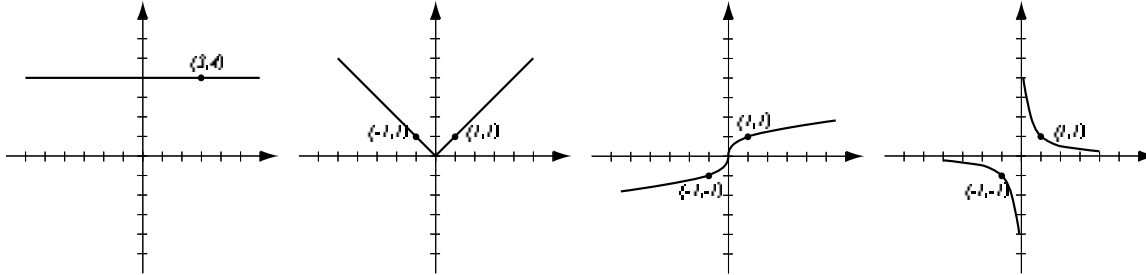
11. (14pts) A truck starts driving eastward from Murray along state route 80. A car driving 11mph faster starts along the same route half an hour afterwards. After the car drives two and a half hours, it catches up with the truck.

a) How fast are the truck and the car?

b) How far from Murray are they when the car catches up with the truck?

Bonus (10pts) Bruce has a total of \$3000 invested in two accounts, one bearing 3% and the other 4% interest. He notices that if he reversed the amounts invested in each account, he would have \$7 more in interest over a year. How much is invested in each account?

1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



2. (20pts) Let $f(x) = \sqrt{12 - 3x}$, $g(x) = \frac{1}{\sqrt{4x + 11}}$.

Find the following (simplify where possible):

$$(f - g)(0) =$$

$$(fg)(-2) =$$

$$\frac{f}{g}(x) =$$

$$(g \circ f)(1) =$$

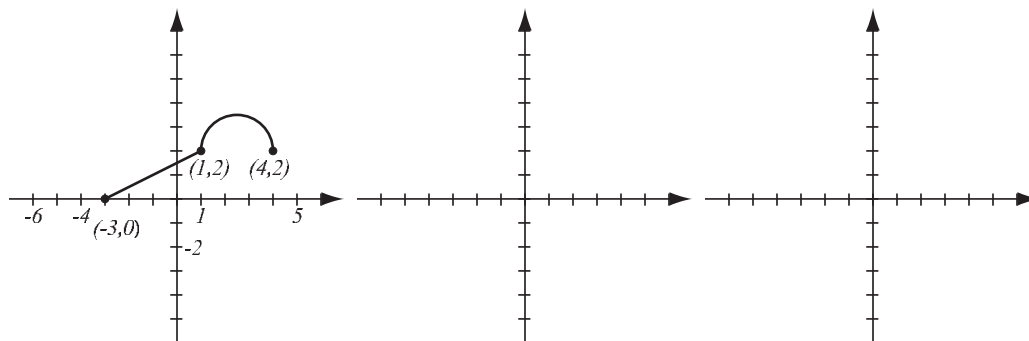
$$(f \circ g)(x) =$$

The domain of $f + g$ in interval notation

3. (6pts) Consider the function $h(x) = \sqrt{x^2 - 2x + 3}$ and find **two** different solutions to the following problem: find functions f and g so that $h(x) = f(g(x))$, where neither f nor g are the identity function.

4. (6pts) Write the equation for the function whose graph has the following characteristics:
 a) shape of $y = x^2$, stretched horizontally by factor 2.
 b) shape of $y = \sqrt[3]{x}$, shifted up 5 units, then reflected over the x axis.

5. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $f(-x) - 2$ and $3f(x + 2)$ and label all the relevant points.



6. (8pts) Sketch the graph of the piecewise-defined function:

$$f(x) = \begin{cases} -x - 5, & \text{if } -4 \leq x \leq -1 \\ x - 2, & \text{if } -1 < x \leq 3 \end{cases}$$

7. (8pts) Find the values of the piecewise-defined function.

$$f(x) = \begin{cases} x^2 - 2, & \text{if } -2 < x < 1 \\ 3x + 1, & \text{if } 1 \leq x \leq 4 \\ 5\sqrt{x}, & \text{if } 4 < x \end{cases}$$

$$f(9) =$$

$$f(0) =$$

$$f(-4) =$$

$$f(3) =$$

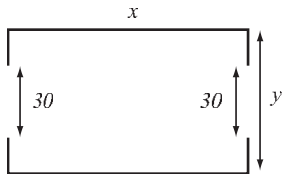
8. (20pts) Let $f(x) = x^5 - 7x^3 + 6x$ (answer with 6 decimal points accuracy).

- Use your graphing calculator to accurately draw the graph of f (on paper!). Indicate units on the axes.
- Determine algebraically whether the function is odd, even, or neither.
- Verify your conclusion from b) by stating symmetry.
- Find the local maxima and minima for this function. If there is symmetry, use it to reduce the work here.
- State the intervals where the function is increasing and where it is decreasing.

9. (14pts) A trucking company wishes to build a service garage for trucks that is to have area 8000 square feet, and has openings on two sides that are 30 feet wide (see picture). To minimize cost, the total length of walls has to be as small as possible.

a) Express the total length of walls of the garage as a function of the length of one of the sides x . What is the domain of this function?

b) Graph the function in order to find the minimum. What are the dimensions of the garage that has the smallest total wall length?



Bonus. (10pts) Let $f(x) = x^2 - 8x + 18$ and $g(x) = 4 - \sqrt{x - 2}$. Find the functions $(f \circ g)(x)$ and $(g \circ f)(x)$ and simplify until they are very simple.

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Name: _____
Show all your work!

Simplify, so that the answer is in form $a + bi$.

1. (5pts) $i(3 + 4i) - (1 + i)^2 =$

2. (5pts) $\frac{2 - i}{5 - 2i} =$

3. (4pts) Simplify and justify your answer.

$i^{43} =$

4. (6pts) Solve the equation by completing the square.

$x^2 - 8x + 22 = 0$

5. (6pts) Solve the inequality. Write the solution in interval form.

$|x + 4| < 3$

6. (6pts) Let $P(x)$ be a polynomial of degree 4.

a) Draw a graph of P that has the maximal number of x -intercepts.

b) Draw a graph of P that has exactly 3 x -intercepts and the maximal number of turning points.

7. (12pts) The quadratic function $f(x) = 4x^2 - 12x - 7$ is given. Do the following without using the calculator.

- Find the x - and y -intercepts of its graph, if any.
- Find the vertex of the graph.
- Sketch the graph of the function.

Solve the equations:

8. (8pts) $\frac{x}{x+7} + \frac{13x+21}{x^2+4x-21} = \frac{6}{x-3}$

9. (8pts) $5 + \sqrt{57 - 14x} = x$

10. (14pts) The polynomial $f(x) = (x + 3)^2(x - 2)^2$ is given.

a) What is the end behavior of the polynomial?

b) List all the zeros and their multiplicities. Find the y -intercept.

c) Use the graphing calculator along with a) and b) to accurately sketch the graph of f (yes, on paper!).

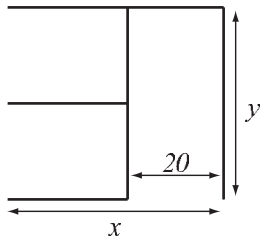
d) Find all the turning points (i.e., local maxima and minima).

11. (12pts) In a right triangle, the long side is 1 meter longer than the short one, and the hypotenuse is twice the length of the short side. What is the length of the short side?

12. (14pts) Shanay is building a three-bay car repair shop, with one bay 20 feet wide. She has enough resources to build 180 feet of walls, and her goal is to maximize the total area of the shop.

a) Express the total area of the shop as a function of the length of one of the sides. What is the domain of this function?

b) Graph the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the shop that has the biggest possible total area, and what is the biggest possible total area?



Bonus. (10pts) Write the formula of a degree-4 polynomial whose x -intercepts are 1, 3 and 6, the y intercept is 9, and the graph touches the x -axis at the x -intercept 1.

College Algebra — Exam 4
MAT 140, Spring 2021 — D. Ivanšić

Name: _____
Show all your work!

1. (8pts) Evaluate without using the calculator:

$$\log_3 81 = \quad \log_2 \frac{1}{32} = \quad \log_u \sqrt[7]{a^3} = \quad \log_{\sqrt{a}} a^3 =$$

2. (4pts) Use the change-of-base formula and your calculator to find $\log_5 3$ with accuracy 6 decimal places. Show how you obtained your number.

3. (5pts) If $\log_a 4 = u$ and $\log_a 20 = v$, express in terms of u and v :

$$\log_a 80 = \quad \log_a \sqrt{5} =$$

4. (6pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$$\log_3 \left(9x^2 \sqrt[3]{y^8} \right) =$$

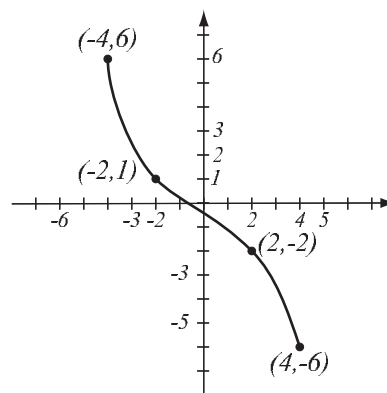
5. (6pts) Write as a single logarithm. Simplify if possible.

$$3 \log_5(x^2 y^5) - 4 \log_5(x^{-2} y^3) =$$

6. (4pts) Simplify.

$$\ln e^{u-v} = \quad 7^{\log_7 14} =$$

7. (6pts) The graph of a function f is given.
- Is this function one-to-one? Justify.
 - If the function is one-to-one, find the graph of f^{-1} , labeling the relevant points, and showing any asymptotes.



8. (9pts) Let $f(x) = \frac{3x + 4}{4x + 5}$.
- Find the formula for f^{-1} .
 - Find the range of f^{-1} .

9. (6pts) Using transformations, draw the graph of $f(x) = -\log_3(x + 2)$. Explain how you transform the graph of a basic function in order to get the graph of f . Indicate at least one point on the graph and any asymptotes.

10. (3pts) Find the domain of the function $f(x) = \ln(5x - 14)$ and write it in interval notation.

11. (9pts) How much needs to be deposited in an account bearing 3.2% interest, compounded quarterly, so that there is \$5,000 in the account after 7 years?

Solve the equations.

12. (6pts) $25^{x+1} = \left(\frac{1}{125}\right)^{x-1}$

13. (8pts) $4^{x-2} = 7^{9x+1}$

14. (8pts) $10^{2x} - 8 \cdot 10^x - 20 = 0$

15. (12pts) According to census data, the population of Kentucky 4,339,367 in 2010 and 4,505,836 in 2020. Assume that it has grown according to the formula $P(t) = P_0e^{kt}$.

a) Find k and write the function that describes the population at time t years since 2010. Graph it on paper.

b) Find the predicted population in the year 2028.

Bonus (10pts) Let $f(x) = \frac{e^x - e^{-x}}{2}$ and $g(x) = x + \sqrt{x^2 + 1}$. Show that $f(g(x)) = x$, which tells you that g and f are inverses to each other.