College Algebra — Exam 4 **MAT 140, Spring 2021 — D. Ivanšić**

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Show all your work!

1. (8pts) Evaluate without using the calculator:

$\log_3 81 =$ 4 $\qquad \log_2 \frac{1}{32} =$ -5 $\qquad \log_u \sqrt[7]{u^3} =$ $\frac{3}{7}$ $\qquad \log_{\sqrt{a}} a^3 =$ 6

$3^? = 81 \qquad 2^? = \frac{1}{32} = \frac{1}{2^5} = 2^{-5} \qquad u^? = \sqrt[7]{u^3} = u^{\frac{3}{7}} \qquad \sqrt{a}^{\,?} = a^3 \quad (a^{\frac{1}{2}})^? = a^3 \quad ? = 6$

2. (4pts) Use the change-of-base formula and your calculator to find $\log_5 3$ with accuracy 6 decimal places. Show how you obtained your number.

$\log_5 3 = \frac{\ln 3}{\ln 5} = 0.682606$

3. (5pts) If $\log_a 4 = u$ and $\log_a 20 = v$, express in terms of u and v:

$\log_a 80 = \log_a (4 \cdot 20)$
$= \log_a 4 + \log_a 20$
$= u + v$

$\log_a \sqrt{5} = \log_a 5^{\frac{1}{2}} = \frac{1}{2} \log_a 5$
$= \frac{1}{2} \log_a \frac{20}{4} = \frac{1}{2} (\log_a 20 - \log_a 4)$
$= \frac{1}{2} (v - u)$

4. (6pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$\log_3 \left(9x^2 \sqrt[3]{y^8}\right) = \log_3 9 + \log_3 x^2 + \log_3 y^{\frac{8}{3}}$
$= 2 + 2 \log_3 x + \frac{8}{3} \log_3 y$

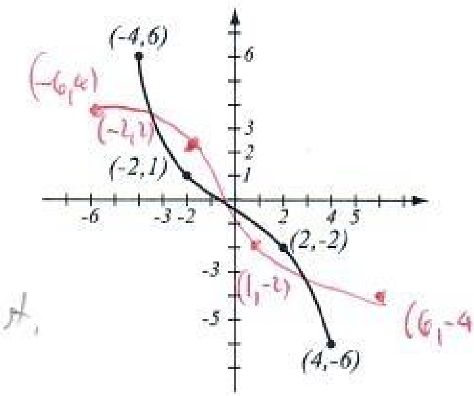
5. (6pts) Write as a single logarithm. Simplify if possible.

$3 \log_5(x^2y^5) - 4 \log_5(x^{-2}y^3) = \log_5 (x^2y^5)^3 - \log_5 (x^{-2}y^3)^4$
$= \log_5 \frac{(x^2y^5)^3}{(x^{-2}y^3)^4} = \log_5 \frac{x^6 y^{15}}{x^{-8} y^{12}} = \log_5 (x^{14} y^3)$

6. (4pts) Simplify.

$\ln e^{u-v} =$ $u - v$ $\qquad 7^{\log_7 14} =$ 14

7. (6pts) The graph of a function f is given.
a) Is this function one-to-one? Justify.
b) If the function is one-to-one, find the graph of f^{-1}, labeling the relevant points, and showing any asymptotes.



a) Yes - it passes the horizontal line test.

8. (9pts) Let $f(x) = \dfrac{3x+4}{4x+5}$.
a) Find the formula for f^{-1}.
b) Find the range of f^{-1}.

$$y = \frac{3x+4}{4x+5}$$

$$y(4x+5) = 3x+4$$

$$4yx + 5y = 3x + 4$$

$$4yx - 3x = 4 - 5y$$

$$(4y-3)x = 4 - 5y$$

$$x = \frac{4-5y}{4y-3} = f^{-1}(y)$$

Range f^{-1} = domain of f

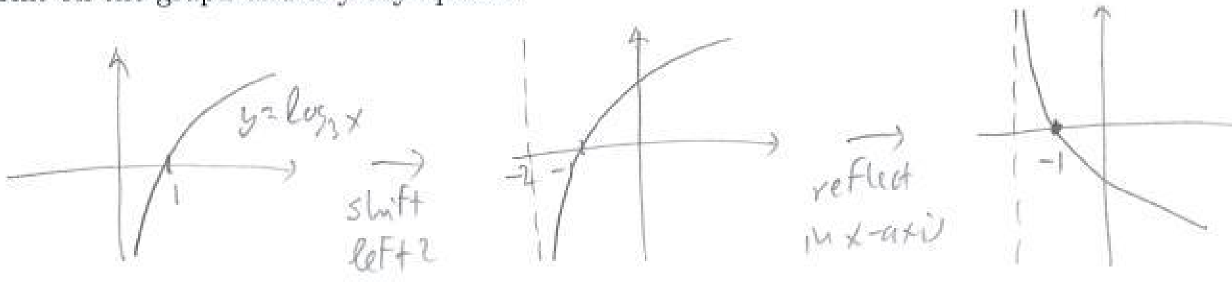
Can't have $4x+5=0$

$$4x = -5$$

$$x = -\frac{5}{4}$$

$$\left(-\infty, -\tfrac{5}{4}\right) \cup \left(-\tfrac{5}{4}, \infty\right)$$

9. (6pts) Using transformations, draw the graph of $f(x) = -\log_3(x+2)$. Explain how you transform the graph of a basic function in order to get the graph of f. Indicate at least one point on the graph and any asymptotes.



10. (3pts) Find the domain of the function $f(x) = \ln(5x - 14)$ and write it in interval notation.

Must have $5x - 14 > 0$

$5x > 14$

$x > \frac{14}{5}$

$\left(\frac{14}{5}, \infty\right)$

11. (9pts) How much needs to be deposited in an account bearing 3.2% interest, compounded quarterly, so that there is \$5,000 in the account after 7 years?

$A = P\left(1 + \frac{r}{4}\right)^{4t}$

$5000 = P \cdot \left(1 + \frac{0.032}{4}\right)^{4 \cdot 7}$

$5000 = P \cdot 1.249956$

$P = \frac{5000}{1.24\ldots} = 4000.14$

Solve the equations.

12. (6pts) $25^{x+1} = \left(\frac{1}{125}\right)^{x-1}$

$(5^2)^{x+1} = \left(\frac{1}{5^3}\right)^{x-1}$

$5^{2x+2} = 5^{-3(x-1)}$

$2x + 2 = -3x + 3$

$5x = 1, \quad x = \frac{1}{5}$

13. (8pts) $4^{x-2} = 7^{9x+1}$ $\mid \ln$

$\ln 4^{x-2} = \ln 7^{9x+1}$

$(x-2)\ln 4 = (9x+1)\ln 7$

$x \ln 4 - 2\ln 4 = 9x \ln 7 + \ln 7$

$x \ln 4 - 9x \ln 7 = 2\ln 4 + \ln 7$

$(\ln 4 - 9\ln 7)x = 2\ln 4 + \ln 7$

$x = \frac{2\ln 4 + \ln 7}{\ln 4 - 9\ln 7} = -0.292586$

14. (8pts) $10^{2x} - 8 \cdot 10^x - 20 = 0$

$(10^x)^2 - 8 \cdot 10^x - 20 = 0$

$u = 10^x$ $\quad u^2 - 8u - 20 = 0$

$(u - 10)(u + 2) = 0$

$u = 10, -2$

$10^x = 10, \quad 10^x = -2$ no sol. $(10^x > 0)$

$x = 1$

15. (12pts) According to census data, the population of Kentucky 4,339,367 in 2010 and 4,505,836 in 2020. Assume that it has grown according to the formula $P(t) = P_0 e^{kt}$.
a) Find k and write the function that describes the population at time t years since 2010. Graph it on paper.
b) Find the predicted population in the year 2028.

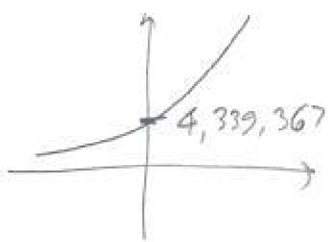
a) $P(t) = 4{,}339{,}367\, e^{kt}$

$4{,}505{,}836 = P(10) = 4{,}339{,}367\, e^{k\cdot 10}$

$e^{10k} = \dfrac{4{,}505{,}836}{4{,}339{,}367}$ $\quad | \ln$

$10k = \ln\left(\text{———}\right)$

$k = \dfrac{\ln \dfrac{4{,}505{,}836}{4{,}339{,}367}}{10} = 0.0037645$



b) 2028 is $t = 18$

$P(18) = 4{,}339{,}367 \cdot e^{0.0037\ldots \cdot 18}$

$= 4{,}643{,}598$ people

Bonus (10pts) Let $f(x) = \dfrac{e^x - e^{-x}}{2}$ and $g(x) = \overset{\ln}{} x + \sqrt{x^2+1}$. Show that $f(g(x)) = x$, which tells you that g and f are inverses to each other.

$f(g(x)) = f\left(\ln(x+\sqrt{x^2+1})\right) = \dfrac{e^{\ln(x+\sqrt{x^2+1})} - e^{-\ln(x+\sqrt{x^2+1})}}{2} = \dfrac{x+\sqrt{x^2+1} - (x+\sqrt{x^2+1})^{-1}}{2}$

$= \frac{1}{2}\left(x+\sqrt{x^2+1} - \dfrac{1}{x+\sqrt{x^2+1}}\right) = \frac{1}{2}\, \dfrac{(x+\sqrt{x^2+1})^2 - 1}{x+\sqrt{x^2+1}} = \dfrac{x^2 + 2\cdot x\sqrt{x^2+1} + \sqrt{x^2+1}^{\,2} - 1}{2(x+\sqrt{x^2+1})}$

$= \dfrac{x^2 + 2x\sqrt{x^2+1} + x^2 + \cancel{1} - \cancel{1}}{2(x+\sqrt{x^2+1})} = \dfrac{2x^2 + 2x\sqrt{x^2+1}}{2(x+\sqrt{x^2+1})} = \dfrac{\cancel{2}x(\cancel{x+\sqrt{x^2+1}})}{\cancel{2}(\cancel{x+\sqrt{x^2+1}})} = x$