College Algebra — Exam 3
MAT 140, Spring 2021 — D. Ivanšić

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Show all your work!

Simplify, so that the answer is in form $a + bi$.

1. (5pts) $i(3+4i) - (1+i)^2 = 3i + \underbrace{4i^2}_{-4} - (1^2 + 2\cdot 1\cdot i + \underbrace{i^2}_{=-1}) = 3i - 4 - 2i$

$= i - 4$

2. (5pts) $\dfrac{2-i}{5-2i} = \dfrac{2-i}{5-2i}\cdot\dfrac{5+2i}{5+2i} = \dfrac{10+4i-5i-2i^2}{5^2-(2i)^2} = \dfrac{12-i}{25-4i^2} = \dfrac{12-i}{29}$

3. (4pts) Simplify and justify your answer.

$i^{43} = i^{40}\cdot i^3 = (i^4)^{10}\cdot i^3 = 1^{10}\cdot i^3 = i^3 = -i$

4. (6pts) Solve the equation by completing the square.

$x^2 - 8x + 22 = 0 \quad | + 4^2$

$x^2 - 2\cdot x\cdot 4 + 4^2 + 22 = 4^2 \quad | -22$

$(x-4)^2 = -6$

$x - 4 = \pm\sqrt{6}\,i$

$x = 4 \pm \sqrt{6}\,i$

5. (6pts) Solve the inequality. Write the solution in interval form.

$|x+4| < 3$

$|x-(-4)| < 3$

dist. from x to -4 < 3

$-3 \quad +3$

$-7 \quad -4 \quad -1$

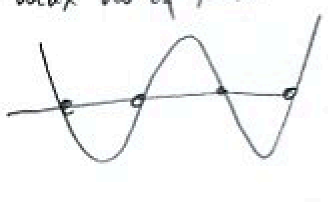
$(-7, -1)$

6. (6pts) Let $P(x)$ be a polynomial of degree 4.

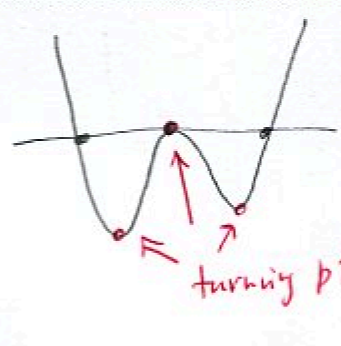
a) Draw a graph of P that has the maximal number of x-intercepts.

b) Draw a graph of P that has exactly 3 x-intercepts and the maximal number of turning points.

a) max no of x-int $= 4$



b) max no of turning pts $= 3$



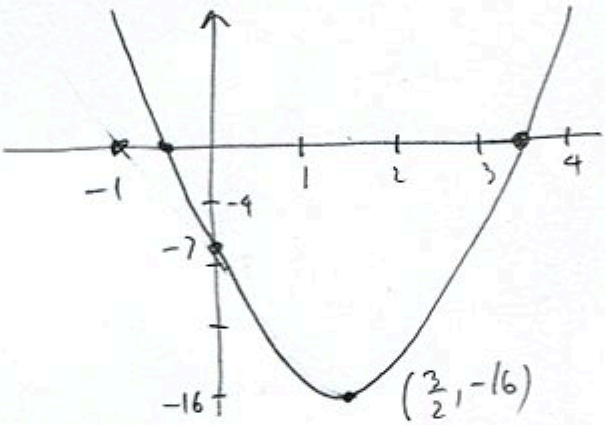
turning pts.

7. (12pts) The quadratic function $f(x) = 4x^2 - 12x - 7$ is given. Do the following without using the calculator.

a) Find the x- and y-intercepts of its graph, if any.

b) Find the vertex of the graph.

c) Sketch the graph of the function.



a) y-int: $f(0) = -7$

x-int: $4x^2 - 12x - 7 = 0$ $\quad 144+112$

$$x = \frac{-(-12) \pm \sqrt{(-12)^2 - 4\cdot 4\cdot(-7)}}{2\cdot 4}$$

$$= \frac{12 \pm \sqrt{256}}{8} = \frac{12 \pm 16}{8} = \frac{28}{8}, -\frac{4}{8}$$

$$= \frac{7}{2}, -\frac{1}{2}$$

vertex: $h = -\frac{b}{2a} = -\frac{-12}{2\cdot 4} = \frac{12}{8} = \frac{3}{2}$

$k = f(\frac{3}{2}) = 4\cdot\frac{9}{4} - 12\cdot\frac{3}{2} - 7 = 9 - 18 - 7$

$= -16$

Solve the equations:

8. (8pts) $\dfrac{x}{x+7} + \dfrac{13x+21}{x^2+4x-21} = \dfrac{6}{x-3}$ $\quad \Big| \cdot (x+7)(x-3)$

$\quad (x+7)(x-3)$

$$\frac{x}{\cancel{x+7}}\cancel{(x+7)}(x-3) + \frac{13x+21}{\cancel{(x+7)(x-3)}}\cancel{(x+7)(x-3)} = \frac{6}{\cancel{x-3}}(x+7)\cancel{(x-3)}$$

$x(x-3) + 13x + 21 = 6(x+7)$

$x^2 - 3x + 13x + 21 = 6x + 42 \quad | -6x - 42$

$x^2 + 4x - 21 = 0$

$(x+7)(x-3) = 0$

$x = -7, 3 \leftarrow$ both give 0 in denominator

No solution.

9. (8pts) $5 + \sqrt{57 - 14x} = x$

$\sqrt{57-14x} = x - 5 \quad |^2$

$57 - 14x = x^2 - 2\cdot x\cdot 5 + 5^2$

$57 - 14x = x^2 - 10x + 25 \quad | +14x - 57$

$x^2 + 4x - 32 = 0$

$(x+8)(x-4) = 0$

$x = -8, 4$

Check: $5 + \sqrt{57+112} = -8$

$5 + 13 = -8$ no

$5 + \sqrt{57-56} = 4$

$5 + 1 = 4$ no

No solution

10. (14pts) The polynomial $f(x) = (x+3)^2(x-2)^2$ is given.
a) What is the end behavior of the polynomial?
b) List all the zeros and their multiplicities. Find the y-intercept.
c) Use the graphing calculator along with a) and b) to accurately sketch the graph of f (yes, on paper!).
d) Find all the turning points (i.e., local maxima and minima).

a) $(x\_)^2 \cdot (x\_\_)^2 = x^2 \cdot x^2 = x^4$

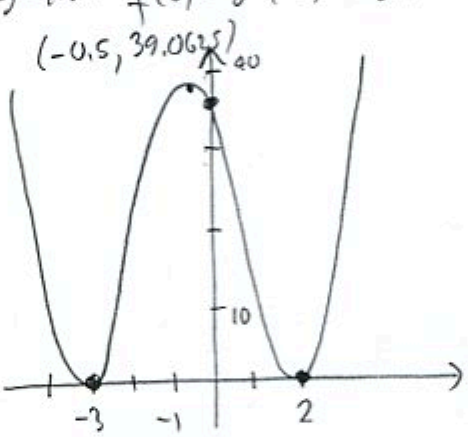
like x^4 $\cup$

b)

zero	-3	2
mult	2	2

y-int: $f(0) = 3^2 \cdot (-2)^2 = 36$

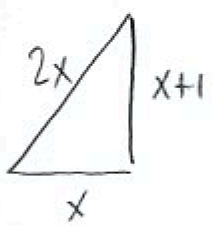
c)



d) Turning pts are:

$(-3, 0)$, $(-0.5, 39.0625)$, $(2, 0)$

11. (12pts) In a right triangle, the long side is 1 meter longer than the short one, and the hypothenuse is twice the length of the short side. What is the length of the short side?



$$x^2 + (x+1)^2 = (2x)^2$$
$$x^2 + x^2 + 2 \cdot x \cdot 1 + 1 = 4x^2$$
$$2x^2 + 2x + 1 = 4x^2$$
$$2x^2 - 2x - 1 = 0$$
$$x = \frac{-(-2) \pm \sqrt{(-2)^2 - 4 \cdot 2 \cdot (-1)}}{2 \cdot 2}$$
$$= \frac{2 \pm \sqrt{12}}{4} = \frac{2 \pm 2\sqrt{3}}{4}$$
$$= \frac{1 \pm \sqrt{3}}{2}$$

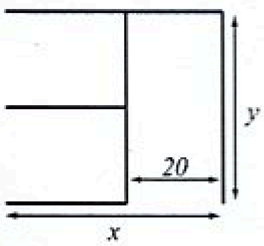
Since $\frac{1-\sqrt{3}}{2} < 0$, it is not a sol., so

$x = \frac{1+\sqrt{3}}{2} = 1.366025$

12. (14pts) Shanay is building a three-bay car repair shop, with one bay 20 feet wide. She has enough resources to build 180 feet of walls, and her goal is to maximize the total area of the shop.

a) Express the total area of the shop as a function of the length of one of the sides. What is the domain of this function?

b) Graph the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the shop that has the biggest possible total area, and what is the biggest possible total area?



a) $A = xy = x\left(-\frac{3}{2}x + 110\right) = -\frac{3}{2}x^2 + 110x$

$x + 2(x-20) + 2y = 180$

$3x - 40 + 2y = 180$

$2y = -3x + 220$

$y = -\frac{3}{2}x + 110$

Domain:

Must have:

$x \geq 20$

$y \geq 0$

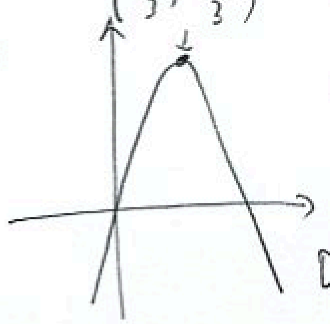
$-\frac{3}{2}x + 110 \geq 0$

$-\frac{3}{2}x \geq -110 \quad \Big| \cdot -\frac{2}{3}$

$x \leq \frac{220}{3} = 73\frac{1}{3}$

$\left[20, \frac{220}{3}\right]$ = domain

b) $\left(\frac{110}{3}, \frac{6050}{3}\right)$



$h = -\frac{b}{2a} = -\frac{110}{-2 \cdot \frac{3}{2}} = \frac{110}{3}$

$k = \frac{110}{3} \cdot \left(-\frac{3}{2} \cdot \frac{110}{3} + 110\right) = \frac{110}{3} \cdot 55$

$\frac{6050}{3} = 2016.666667$

$-\frac{3}{2} \cdot \frac{110}{3} + 110$

Dimensions are: $\frac{110}{3} \times 55$

Max area is $\frac{6050}{3} = 2016.666667$

Bonus. (10pts) Write the formula of a degree-4 polynomial whose x-intercepts are 1, 3 and 6, the y intercept is 9, and the graph touches the x-axis at the x-intercept 1.

$f(x) = a(x-1)^2(x-3)(x-6)$ exp. has to be even since graph touches x-axis at x-int. 1

can't be more than 2, since degree of polynomial is 4

$9 = f(0) = a \cdot (0-1)^2(0-3)(0-6)$

$9 = a \cdot 18$

$a = \frac{9}{18} = \frac{1}{2}$

$f(x) = \frac{1}{2}(x-1)^2(x-3)(x-6)$