College Algebra — Exam 2 MAT 140, Spring 2021 — D. Ivanšić

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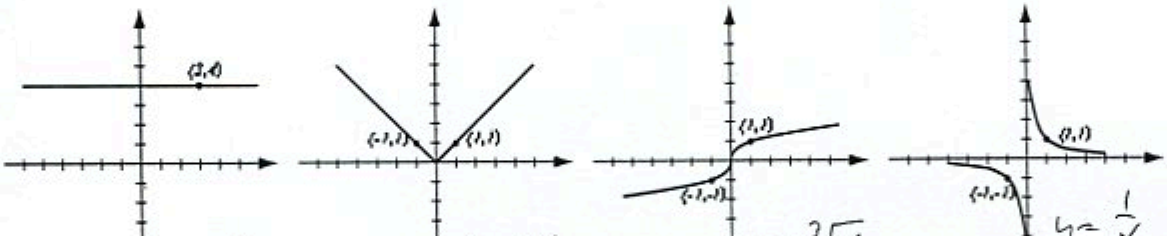
Show all your work!

1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



$y=4$ $\quad$ $y=|x|$ $\quad$ $y=\sqrt[3]{x}$ $\quad$ $y=\frac{1}{x}$

2. (20pts) Let $f(x)=\sqrt{12-3x}$, $g(x)=\dfrac{1}{\sqrt{4x+11}}$.

Find the following (simplify where possible):

$(f-g)(0) = f(0)-g(0)=\sqrt{12}-\frac{1}{\sqrt{11}}$

$(fg)(-2) = f(-2)\cdot g(-2)$
$= \sqrt{18}\cdot\frac{1}{\sqrt{3}} = \frac{\sqrt{18}}{\sqrt{3}} = \sqrt{\frac{18}{3}} = \sqrt{6}$

$\frac{f}{g}(x) = \frac{f(x)}{g(x)} = \frac{\sqrt{12-3x}}{\frac{1}{\sqrt{4x+11}}} = \sqrt{12-3x}\cdot\sqrt{4x+11}$

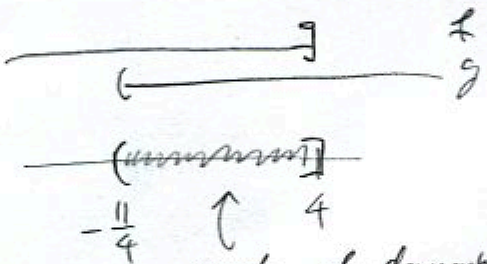
$(g\circ f)(1) = g(f(1)) = g(\sqrt{12-3})$
$= g(3) = \frac{1}{\sqrt{12+11}} = \frac{1}{\sqrt{23}}$

$(f\circ g)(x) = f(g(x)) = f\left(\frac{1}{\sqrt{4x+11}}\right) = \sqrt{12-3\cdot\frac{1}{\sqrt{4x+11}}}$
$= \sqrt{12-\frac{3}{\sqrt{4x+11}}}$

The domain of $f+g$ in interval notation

Domain f: must have $12-3x\geq 0$
$3x\leq 12$
$x\leq 4$

Domain g: must have $4x+11>0$
$4x>-11$
$x>-\frac{11}{4}$



$\left(-\frac{11}{4}, 4\right]$ is domain of $f+g$

3. (6pts) Consider the function $h(x) = \sqrt{x^2 - 2x + 3}$ and find **two** different solutions to the following problem: find functions f and g so that $h(x) = f(g(x))$, where neither f nor g are the identity function.

$g(x) = x^2 - 2x + 3$ $\qquad$ $g(x) = x^2 - 2x$

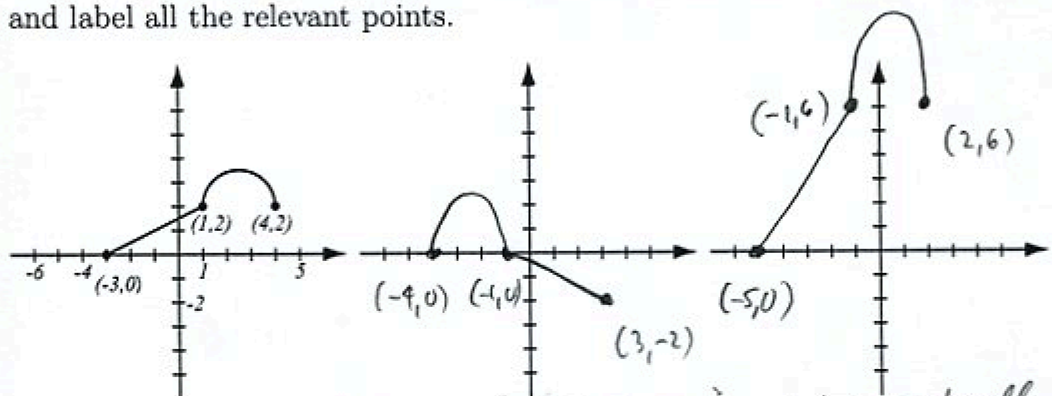
$f(x) = \sqrt{x}$ $\qquad$ $f(x) = \sqrt{x+3}$

4. (6pts) Write the equation for the function whose graph has the following characteristics:
a) shape of $y = x^2$, stretched horizontally by factor 2.
b) shape of $y = \sqrt[3]{x}$, shifted up 5 units, then reflected over the x axis.

a) $y = x^2 \xrightarrow[\text{replace } x \text{ by } \frac{1}{2}x]{} y = \left(\frac{1}{2}x\right)^2 = \frac{1}{4}x^2$

b) $y = \sqrt[3]{x} \xrightarrow[\text{add } 5]{} y = \sqrt[3]{x} + 5 \xrightarrow[\text{take opposite of formula}]{} y = -\sqrt[3]{x} - 5$

5. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $f(-x) - 2$ and $3f(x+2)$ and label all the relevant points.



reflect in y-axis: $x \mapsto -x$
shift down 2 $y \mapsto y-2$

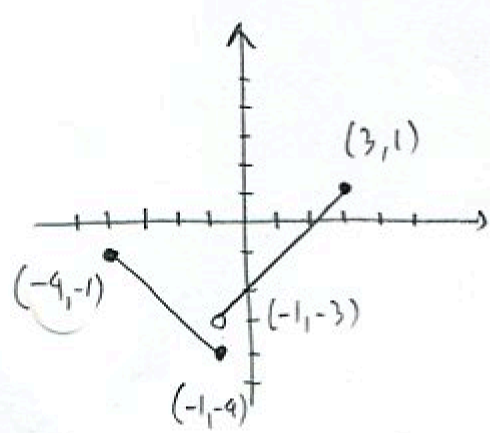
stretch vertically, factor = 3: $y \mapsto 3y$
shift left 2 $\qquad$: $x \mapsto x-2$

6. (8pts) Sketch the graph of the piecewise-defined function:

$$f(x) = \begin{cases} -x - 5, & \text{if } -4 \le x \le -1 \\ x - 2, & \text{if } -1 < x \le 3 \end{cases}$$

x	$-x-5$
-4	-1
-1	-4

x	$x-2$
-1	-3
3	1



7. (8pts) Find the values of the piecewise-defined function.

$$f(x) = \begin{cases} x^2 - 2, & \text{if } -2 < x < 1 \\ 3x + 1, & \text{if } 1 \le x \le 4 \\ 5\sqrt{x}, & \text{if } 4 < x \end{cases}$$

$f(9) =$ $5\sqrt{9} = 15$

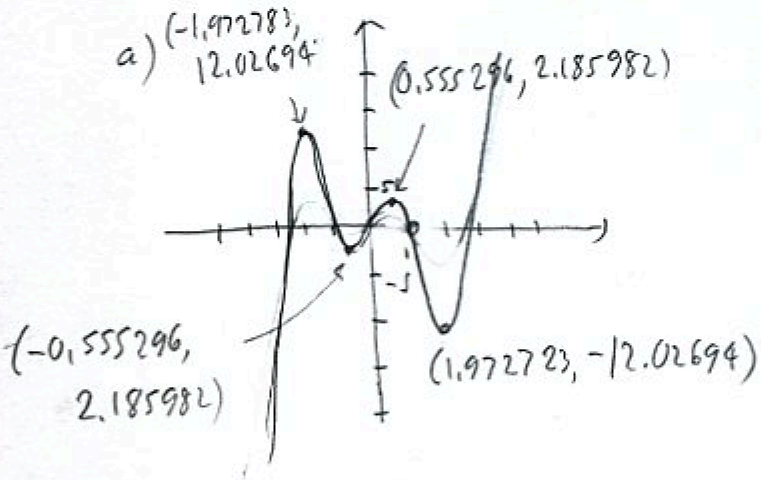
$f(0) =$ $0^2 - 2 = -2$

$f(-4) =$ not defined

$f(3) =$ $3 \cdot 3 + 1 = 10$

8. (20pts) Let $f(x) = x^5 - 7x^3 + 6x$ (answer with 6 decimal points accuracy).
a) Use your graphing calculator to accurately draw the graph of f (on paper!). Indicate units on the axes.
b) Determine algebraically whether the function is odd, even, or neither.
c) Verify your conclusion from b) by stating symmetry.
d) Find the local maxima and minima for this function. If there is symmetry, use it to reduce the work here.
e) State the intervals where the function is increasing and where it is decreasing.

a)



b) $f(-x) = (-x)^5 - 7(-x)^3 + 6(-x)$
$= -x^5 + 7x^3 - 6x = -f(x)$
odd Function

c) graph is symmetric wrt origin

d) $f(-1,972723) = 12.02694$
$f(0,555296) = 2.185982$
} are local maxes

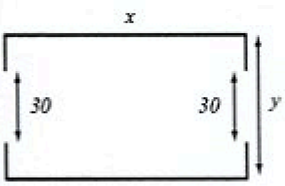
$f(-0,555296) = 2.185982$
$f(1.972723) = -12.02694$
} are local mins

e) Increasing on $(-\infty, -1,972783)$ and $(-0,555296, 0,555296)$ and $(1.972723, \infty)$
Decreasing on $(-1.972783, -0,555296)$ and $(0,555296, 1,972723)$

9. (14pts) A trucking company wishes to build a service garage for trucks that is to have area 8000 square feet, and has openings on two sides that are 30 feet wide (see picture). To minimize cost, the total length of walls has to be as small as possible.

a) Express the total length of walls of the garage as a function of the length of one of the sides x. What is the domain of this function?

b) Graph the function in order to find the minimum. What are the dimensions of the garage that has the smallest total wall length?

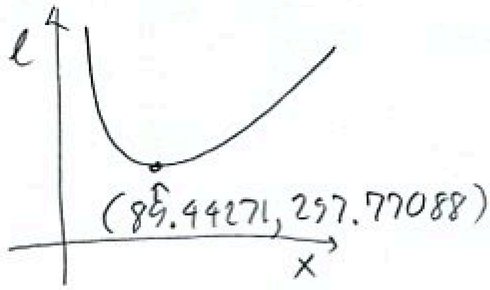


a) $\ell = 2x + 2(y-30) = 2x + 2y - 60 = 2x + \frac{16000}{x} - 60 = \ell(x)$

$x \cdot y = 8000$, so $y = \frac{8000}{x}$

Domain: Must have: $x > 0$, $y \geq 30$ $\quad \frac{8000}{x} \geq 30$, $\quad x \leq \frac{8000}{30} = 266.666667$

Domain: $\left(0, \frac{800}{3}\right)$



Dimensions: 89.44271×89.44271

Min wall length: 297.77088

Bonus. (10pts) Let $f(x) = x^2 - 8x + 18$ and $g(x) = 4 - \sqrt{x-2}$. Find the functions $(f \circ g)(x)$ and $(g \circ f)(x)$ and simplify until they are very simple.

$$(f \circ g)(x) = f(4 - \sqrt{x-2}) = (4 - \sqrt{x-2})^2 - 8(4 - \sqrt{x-2}) + 18$$

$$= 4^2 - \cancel{2 \cdot 4 \cdot \sqrt{x-2}} + \sqrt{x-2}^2 - 32 + \cancel{8\sqrt{x-2}} + 18$$

$$= 16 + x - 2 - 32 + 18 = x$$

$$(g \circ f)(x) = g(x^2 - 8x + 18) = 4 - \sqrt{x^2 - 8x + 18 - 2} = 4 - \sqrt{\underbrace{x^2 - 8x + 16}_{(x-4)(x-4)}} = 4 - \sqrt{(x-4)^2}$$

$$= 4 - (x-4) = 8 - x$$