

**Example.** Draw graphs of power functions  $x^n$  for the exponents given.

$$x^2, x^4, x^6$$

$$x^3, x^5, x^7$$

Graphs of  $x^{\text{even}}$  have the same shape as  $x^2$ , graphs of  $x^{\text{odd}}$  have the same shape as  $x^3$ .

**Definition.** A general *polynomial* is a function of form

$$f(x) = a_n x^n + a_{n-1} x^{n-1} + \dots + a_2 x^2 + a_1 x + a_0$$

Some terminology:  $a_n x^n$  is called the *leading term*,  $a_n$  the *leading coefficient*; if  $a_n \neq 0$ ,  $n$  is the *degree* of the polynomial.

**Example.** Compare the graphs of  $f(x) = 2x^3 - 3x^2 - 5x - 4$  and  $g(x) = 2x^3$ .

**Fact.** For large  $x$  or large negative  $x$ , every polynomial behaves like its leading term  $a_n x^n$ . This is referred to as the *end behavior* of the polynomial. Thus, graphs of polynomials look like one of the four pictures below.

**Example.** Draw the graph of the polynomial  $f(x) = (x - 2)^2(x + 1)(x - 4)$ .

$x$ -intercepts (also called *zeroes*) are 2, -1, 4

their corresponding *multiplicities* are 2, 1, 1 (exponents on factors related to the zeroes)

**Fact.** If  $c$  is a zero of a polynomial  $P(x)$ , then  $x - c$  is a factor of  $P(x)$ , so  $P(x) = (x - c)^k g(x)$ .

**Definition.** If  $(x - c)^{k+1}$  is not a factor of the polynomial  $P(x)$ , but  $(x - c)^k$  is, we say the zero  $c$  has multiplicity  $k$ . Behavior of the graph at a zero depends on its multiplicity in this way:

Multiplicity of $c$	Graph of $P(x)$ at $c$
even	touches $x$ -axis
odd	crosses $x$ -axis

**Example.** Let  $f(x) = (x + 3)^2(x^2 + 7)(1 - x)$ . For the polynomial, find the zeroes and their multiplicities, determine end behavior and use this information to help you sketch the graph of the polynomial.

**Guidelines for graphing a polynomial**  $f(x) = a_nx^n + a_{n-1}x^{n-1} + \dots + a_2x^2 + a_1x + a_0$

- 1) Determine end behavior — for large  $|x|$ , it looks like  $a_nx^n$ .
- 2) Find the  $y$ -intercept, the zeroes ( $x$ -intercepts, there can be at most  $n$  zeroes) and their multiplicities.
- 3) Find the turning points (local minima and maxima, there can be at most  $n - 1$  of them).

**Example.** Use the guidelines to graph the polynomial  $f(x) = x^4 - 4x^3 + 3x^2$ .