Calculus 2 — Exam 4
MAT 308, Fall 2020 — D. Ivanšić

Name: ______________________________

Show all your work!

Find the intervals of convergence for the series below. Don't forget to check the endpoints.

1. (16pts) $\displaystyle\sum_{n=0}^{\infty} 3^n \cdot \sqrt{n} \cdot (x-4)^n$

2. (10pts) $\displaystyle\sum_{n=1}^{\infty} \frac{e^n}{(2n)!} x^n$

3. (6pts) Use a known power series to find the sum:

$$\sum_{n=1}^{\infty} \frac{(-1)^{n-1}}{n3^n} =$$

4. (8pts) Use a known power series to find the limit.

$$\lim_{x \to 0} \frac{e^x - e^{-x} - 2x}{x^3} =$$

5. (14pts) Use geometric series and differentiation to get a power series for $\frac{x^2}{(1-x^2)^2}$. State the interval of convergence (no need to check the endpoints).

6. (12pts) Use known power series to show that $\dfrac{d}{dx}\cos x = -\sin x$.

7. (18pts) Let $f(x) = \sqrt[3]{x}$.
a) Find the 2nd Taylor polynomial for f centered at $a = 8$.
b) Use Taylor's formula to get an estimate of the error $|R_2|$ on the interval $[6.5, 9.5]$. Leave your answer as a fraction.

8. (16pts) Use the known power series for $\cos x$ to find the series representing $\int_0^{\frac{1}{2}} \cos\sqrt{x}\, dx$. (Note that $\cos\sqrt{x}$ does not have an antiderivative that is an elementary function.) Give an approximation of the definite integral with accuracy 10^{-3}. Write the approximation as a sum (you do not have to simplify it).

Bonus (10pts) Find a fraction that is the approximation of $\sqrt{5}$ with accuracy 10^{-2}. Start as below and take advantage of the binomial series.

$$\sqrt{5} = \sqrt{4\left(1+\frac{1}{4}\right)} =$$