Calculus 2 — Exam 3
MAT 308, Fall 2020 — D. Ivanšić

Name: _____

Show all your work!

Find the limits, if they exist.

1. (6pts) $\lim_{n\to\infty} \frac{n}{1.5^n} =$

2. (6pts) $\lim_{n\to\infty} (2 + (-1)^n) =$

3. (10pts) Find the limit. Use the theorem that rhymes with the insects typically found on cats and dogs.

$$\lim_{n\to\infty} \frac{2^n(\sin(17n) + 1)}{3^n}$$

4. (6pts) Write the series using summation notation:

$$\frac{3}{8} - \frac{7}{16} + \frac{11}{32} - \frac{15}{64} + \cdots =$$

5. (12pts) Justify why the series converges and find its sum.

$$\sum_{n=2}^{\infty} \frac{2 \cdot 3^{2n-1}}{2^{4n+3}} =$$

Determine whether the following series converge and justify your answer.

6. (6pts) $\displaystyle\sum_{n=1}^{\infty} \frac{2n-1}{n}$

7. (12pts) $\displaystyle\sum_{n=1}^{\infty} \frac{1+\sqrt[n]{5}}{3n^2}$

8. (20pts) Consider the alternating series $\sum_{n=2}^{\infty}(-1)^n \frac{1}{n+\sqrt{n}}$.

a) Is the series convergent? Justify.

b) Is the series absolutely convergent? Justify.

Determine whether the following series converge using the root or ratio test.

9. (11pts) $\sum_{n=1}^{\infty}(-1)^{n-1} \frac{22^n}{1 \cdot 3 \cdot 5 \cdot \cdots \cdot (2n-1)}$

10. (11pts) $\displaystyle\sum_{n=1}^{\infty} \frac{n^4 + 3n^2 + 1}{2^{3n+1}}$

Bonus. (10pts) Play this game on a basic calculator: enter any positive number, then keep pressing the $\sqrt{\ }$ key. After a while, the display stabilizes at a number. (In case you have never used a basic calculator, pressing $\sqrt{\ }$ immediately returns the square root of the number.)

a) Use a sequence and a limit to explain what is happening.

b) At which number does the display stabilize?