Calculus 2 — Exam 1
MAT 308, Fall 2020 — D. Ivanšić

Name: _____

Show all your work!

Find the following integrals:

1. (7pts) $\int xe^{3x}\,dx =$

2. (7pts) $\int \sin^2 x\,dx =$

Determine whether the following improper integral converges, and, if so, evaluate it. (Calculate directly, comparison would be hard.)

3. (14pts) $\int_1^\infty \frac{\ln x}{x^2}\,dx =$

Use trigonometric substitution to evaluate the following integrals. Don't forget to return to the original variable where appropriate.

4. (14pts) $\displaystyle\int \frac{x^3}{\sqrt{x^2-1}}\,dx =$

5. (14pts) $\displaystyle\int_0^{\frac{3}{2}} \frac{1}{(9-x^2)^{\frac{3}{2}}}\,dx =$

Use the method of partial fractions to find the following integrals.

6. (14pts) $\displaystyle\int \frac{-x^2-3x+2}{(x+1)(x^2+1)}\,dx =$

7. (10pts) Use comparison to determine whether the improper integral $\displaystyle\int_1^\infty \frac{x^2}{x^4+7}\,dx$ converges.

8. (20pts) Suppose we wanted to approximate the number $\ln 4$. We could do it by approximating the integral $\int_1^4 \frac{1}{x}\,dx = \ln 4$, which uses only the four algebraic operations.
a) Write the expression you would use to calculate T_6, the trapezoid rule with 6 subintervals. All the terms need to be explicitly written, do not use f in the sum.
b) Find the error estimate for T_n in general. You will need the second derivative of $\frac{1}{x}$.
c) Estimate the error for T_6.
d) What should n be in order for T_n to give you an error less than 10^{-4}?

Bonus (10pts) On the interval $[1, 3]$, draw a nice big picture of any concave upward function f whose graph is above the x-axis. Then draw the straight-edge shapes whose area is represented by the trapezoid and midpoint approximations T_2 and M_2 for the integral $I = \int_1^3 f(x)\,dx$. Put the numbers I, T_2 and M_2 in increasing order and justify this order precisely with your picture.