Differentiate and simplify where appropriate:

1. (6pts) $\dfrac{d}{dx}\left(8x^3 - c^6 + \sqrt[6]{x^{11}} - \dfrac{2}{x^3}\right) =$

2. (6pts) $\dfrac{d}{dt}\left(t^2 - 1\right)^3\left(t^2 + 1\right)^3 =$

3. (8pts) $\dfrac{d}{dw}\dfrac{\sqrt[3]{w} + \dfrac{1}{\sqrt[3]{w}}}{w + 1} =$

4. (4pts) $\dfrac{d}{d\theta}\ln(\theta\cos\theta) =$

5. (7pts) (This is a known derivative, your job is to verify it here.)

$\dfrac{d}{dx}\ln|x + \sqrt{x^2 + 1}| =$

6. (6pts) $\dfrac{d}{dx}e^{\sqrt{\arctan x}} =$

7. (5pts) Let $f(x) = xe^{2x}$. Take the first four derivatives of f, and try to spot the pattern. What is $f^{(36)}(x)$, the 36th derivative of f? How about $f^{(n)}(x)$?

Find the following limits. Use L'Hospital's rule if needed.

8. (2pts) $\displaystyle\lim_{x\to\infty} \frac{1}{\sqrt{x}} =$

9. (6pts) $\displaystyle\lim_{x\to\infty} \frac{x^2 - 3x + 1}{x^3 - 3x^2 + 6x + 5} =$

10. (8pts) $\displaystyle\lim_{x\to 0} (1 + x^2)^{\frac{1}{x}} =$

Find the following antiderivatives.

11. (7pts) $\displaystyle \int 3x^5 - \frac{5}{1+x^2} + \frac{2}{\sqrt[4]{x^9}} + e^\pi \, dx =$

12. (3pts) $\displaystyle \int (4x+11)^8 \, dx =$

13. (7pts) $\displaystyle \int \frac{\sqrt{x} - \frac{1}{\sqrt{x}}}{x^2} \, dx =$

Use the substitution rule in the following integrals:

14. (7pts) $\displaystyle \int (x+3)\cos(x^2 + 6x - 7) \, dx =$

15. (10pts) $\displaystyle \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \cos x \, 3^{\sin x} \, dx =$

16. (8pts) Consider the integral $\int_3^7 x^2 - 4x - 5 \, dx$.

a) Draw a picture to explain the meaning of the integral.

b) Use the picture to estimate whether the integral is positive or negative.

c) Evaluate the integral to verify your finding in b).

Bonus. (10pts) The rear inside cover of our book claims that

$$\int \frac{\sqrt{a^2 - x^2}}{x^2} \, dx = -\frac{\sqrt{a^2 - x^2}}{x} - \arcsin \frac{x}{a} + C$$

Verify this formula by differentiating.