

College Algebra — Joysheet 1  
MAT 140, Fall 2021 — D. Ivanišić

Name: \_\_\_\_\_

Covers: JIT 6, 7, 12–14, 17      Show all  
your work!

Write interval notation and sketch on the number line.

1. (3pts)  $\{x|x \geq -3\}$

2. (3pts)  $\{x|-2 < x < 5\}$

Solve the equations.

3. (3pts)  $3x - 5 = 5x + 7$

4. (4pts)  $5(2x - 1) - 3x = 7 + 4(x - 3)$

Simplify and write in standard form:

5. (4pts)  $(x + 5)(x^2 - 2x) - 4x^3 + 7x^2 =$

6. (4pts)  $(x^2 + 3x)x - (x + 5)(x - 1) =$

Simplify and write the answer so all exponents are positive:

7. (2pts)  $u^3(3z)^2 =$

8. (2pts)  $\frac{(4x)^4}{x^9} =$

9. (3pts)  $(u^3v^5)^2 u^4v^3 =$

10. (5pts)  $(x^{-3}y^7)^2(5x^3y^{-2})^2 =$

11. (7pts)  $\frac{(2a^2b^{-4})^3}{(6a^{-2}b^3)^2} =$

Factor the following.

12. (4pts)  $x^2 + x - 12 =$

13. (4pts)  $x^2 - 14x + 45 =$

Solve the equations.

14. (6pts)  $x^2 - 2x - 4 = x + 24$

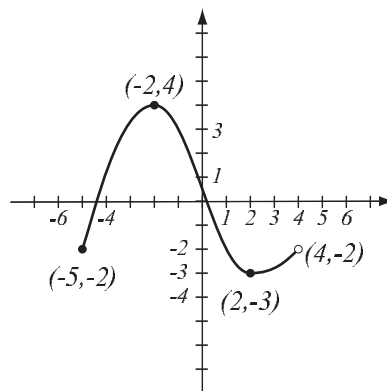
15. (6pts)  $2x^2 + 10x = x^2 + 11$

1. (13pts) Consider the circle with the equation  $(x - 2)^2 + (y - 3)^2 = 10$ .
- a) Algebraically verify that the points  $A = (-1, 2)$ ,  $B = (5, 4)$  and  $C = (1, 6)$  are on the circle. Draw the circle in the coordinate plane (may use the points  $A$ ,  $B$  and  $C$  to help you).
  - c) Take any point  $D$  on the circle other than  $A$ ,  $B$  and draw the triangle  $ABD$ . Does it look like  $ABD$  is a right triangle?
  - d) Because  $AB$  is the diameter of the circle, it is a known theorem that  $ABD$  is a right triangle for any choice of  $D$ . Verify algebraically that  $ABC$  is a right triangle.

2. (8pts) Find the equation of the circle with center  $(-1, -2)$  that contains the point  $(0, 3)$ . Draw the circle.

3. (8pts) Use the graph of the function  $f$  at right to answer the following questions.

- a) Find  $f(-5)$  and  $f(5)$ .
- b) What is the domain of  $f$ ?
- c) What is the range of  $f$ ?
- d) What are the solutions of the equation  $f(x) = 1$ ?



4. (12pts) The function  $f(x) = 2x^2 - 4x - 15$  is given.

a) Use your calculator to accurately its graph. Draw the graph here, and indicate units on the axes.

b) Find all the  $x$ - and  $y$ -intercepts (accuracy: 6 decimal points).

c) State the domain and range.

5. (9pts) Find the domain of each function and write it using interval notation.

$$f(x) = 7 + 3\sqrt{x}$$

$$g(x) = \frac{4x + 7}{x^2 - 12x + 35}$$

6. (10pts) Let  $h(x) = \frac{x^2 - 3x}{2x + 6}$ . Find the following (simplify where appropriate).

$$h(7) =$$

$$h(-3) =$$

$$h(3a) =$$

$$h(x + 4) =$$

1. (6pts) Find the equation of the line (in form  $y = mx + b$ ) whose  $x$ -intercept is -4 and  $y$ -intercept is -1. Draw the graph of the line.

2. (10pts) Find the equation of the line (in form  $y = mx + b$ ) passing through  $(2, 1)$  that is perpendicular to the line  $2x + 3y = 6$ . Draw both lines.

3. (8pts) Draw the quadrangle with vertices  $A = (0, -3)$ ,  $B = (7, -1)$ ,  $C = (6, 3)$  and  $D = (-1, 1)$ .

a) Find the slopes of the sides of the quadrangle.

b) Use slopes to check if the quadrangle is a parallelogram (has two pairs of parallel sides).

c) Is it a rectangle (a parallelogram with adjacent sides perpendicular)? Use slopes to check.

4. (4pts) According to the US census, there were 4,339,367 inhabitants of Kentucky in 2010, and 4,505,836 in 2020. What is the average rate of change of the population of Kentucky from 2010 to 2020? What are the units for the average rate of change?

5. (12pts) The electric bill for a household was \$158.21 in a month when it used 1123 kWh (kilowatt-hours) of electricity. In another month, it used 2546 kWh and was billed \$328.97.
- Assuming that the electricity cost  $C(x)$  is a linear function of the amount of electricity  $x$  used (in kWh), write a formula for  $C(x)$ .
  - What is the cost if no electricity is used during a month? What is the meaning of this number?
  - What is the meaning of the slope in this example?

6. (20pts) A statistician is trying to establish a trend for the annual number of vehicle accident fatalities in the US. In the table,  $D$  is the number of fatalities during year  $x$ , in thousands. Solve the problems below with accuracy 6 decimal points.
- Draw the scatterplot of the data. Does the relationship look linear?
  - Use two points in the scatterplot to get an equation of a line that models the relationship between  $D$  and  $x$ . Draw the line on the graph.
  - Use your calculator to find the “line of best fit” for the data. Draw the line on the graph.
  - Find the coefficient of correlation  $r$ . How strong is the linear relationship between  $x$  and  $D$ ?
  - How many fatalities can one expect in year 2020?

$x$	$D$
1965	47.1
1975	44.5
1985	43.8
1995	41.8
2005	43.5
2015	35.5

Solve the inequalities. Write your solution in interval notation.

1. (5pts)  $0 \leq 8 - 5x < 13$

2. (7pts)  $2x + 3 < 7$  or  $3x - 11 > 7$

3. (6pts) Find the domain of the function in interval notation:  $f(x) = \frac{\sqrt{6 - 3x}}{x + 6}$ .

4. (14pts) Mall masseurs Guido and Jaco offer their services at these costs: Guido charges 80 cents per minute, and Jaco charges \$14 for the first fifteen minutes and then 75 cents per minute for the minutes after fifteen. Assuming you want to get at least a fifteen minute massage, for which number of minutes does Jaco have the better deal? Solve as an inequality.

**5.** (14pts) Jenny traveled to Louisville at 68 miles per hour. On the way back, she took the scenic route and drove 56 miles per hour. The way back was 17 miles longer and took one hour longer to drive than the way to Louisville.

a) How many hours did it take to drive to Louisville?

b) How long is the scenic route from Louisville?

**6.** (14pts) Larry, Moe and Curly share the cost of their lunch, for which the bill was \$23.20. Moe pays half of what Larry pays, and Curly pays \$5 less than Larry and Moe together. How much did each of them contribute to the bill?



1. (10pts) Use your calculator to accurately sketch the graph of the function  $f(x) = x - 4\sqrt[3]{x}$ . Draw the graph here, indicate units on the axes, and solve the problems below with accuracy 6 decimal points.

- Find the local maxima and minima for this function.
- State the intervals where the function is increasing and where it is decreasing.

2. (20pts) Let  $f(x) = \frac{2}{x^2 - 4}$ ,  $g(x) = \sqrt{3x - 1}$ . Find the following (simplify where possible):

$$(f + g)(1) = \qquad (fg)(3) =$$

$$\frac{g}{f}(x) = \qquad (g \circ f)(3) =$$

$$(f \circ g)(x) =$$

The domain of  $(f + g)(x)$  in interval notation

3. (8pts) Consider the function  $h(x) = \frac{7}{x^2 + x - 6}$  and find **two** different solutions to the following problem: find functions  $f$  and  $g$  so that  $h(x) = f(g(x))$ , where neither  $f$  nor  $g$  are the identity function.

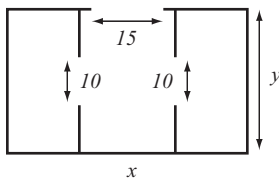
4. (8pts) Sketch the graph of the piecewise-defined function:

$$f(x) = \begin{cases} 2x + 1, & \text{if } x \leq -2 \\ 1 - x, & \text{if } x > -2. \end{cases}$$

5. (14pts) An entrepreneur is designing a simple 3-room store, whose total area is 2500 square feet and which has openings for doors with indicated size. She wishes to minimize construction cost, which is same as minimizing the total length of the walls.

a) Express the total length of the walls of the building as a function of the length of one of the sides  $x$ . What is the domain of this function?

b) Graph the function in order to find the minimum. What are the dimensions of the block for which the total length of the walls is minimal? What is the minimal wall length?



College Algebra — Joysheet 6  
MAT 140, Fall 2021 — D. Ivanišić

Name: \_\_\_\_\_

Covers: 2.4, 2.5

Show all your work!

1. (21pts) For the following functions:

a) determine algebraically whether they are odd, even, or neither

b) use the calculator to draw their graphs here and verify your conclusions by stating symmetry.

$$f(x) = x^3 - 7x^2 + 10x$$

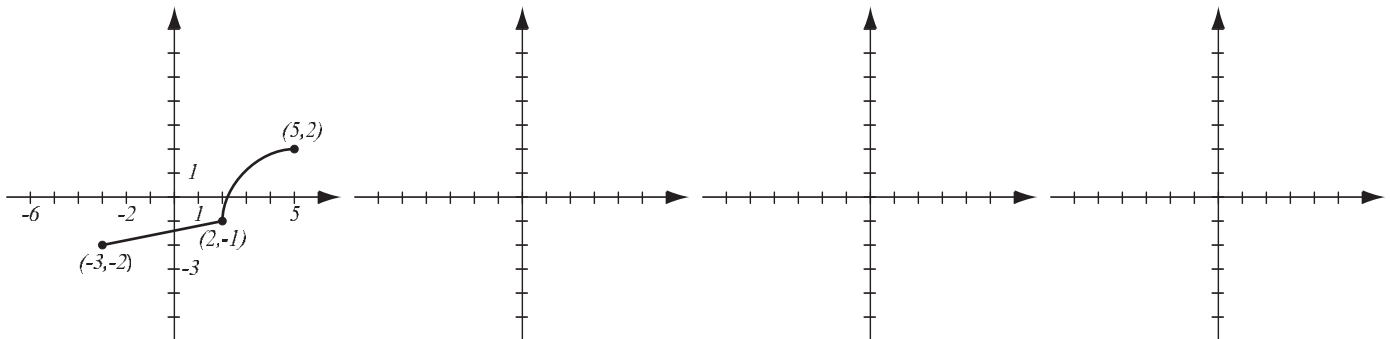
$$g(x) = x^4 - x^2 + 4$$

$$h(x) = -x^3 + 7x$$

2. (16pts) Draw the graphs of  $f(x) = \sqrt{2x} - 5$  and  $g(x) = -2(x+4)^2$  using transformations. Explain how you transform graphs of basic functions in order to get the graphs of  $f$  and  $g$ . Indicate at least two points on each graph.

3. (10pts) Write the equation for the function whose graph has the following characteristics:
- shape of  $y = x^3$ , shifted down 1 unit
  - shape of  $y = \frac{1}{x}$ , stretched horizontally by factor 4, then shifted left 2 units
  - shape of  $y = |x|$ , shifted right 1 unit, then reflected about the  $y$ -axis, then stretched vertically by factor 3.

4. (13pts) The graph of  $f(x)$  is drawn below. On three separate graphs, sketch the graphs of the functions  $f(2x)$ ,  $2f(x + 3)$  and  $f(-x) + 2$  and label all the relevant points.



College Algebra — Joysheet 7  
MAT 140, Fall 2021 — D. Ivanišić

Name: \_\_\_\_\_

*Covers: JIT 13, 15, 21-23 Show all your work!*

Use formulas to expand:

1. (4pts)  $(3x - 4y)(3x + 4y) =$

2. (4pts)  $(a + 3b)^2 =$

3. (5pts)  $(x^4 + y^4)^2 =$

Use the *ac*-method or another method to factor. Show how you got your answer.

4. (5pts)  $2x^2 + x - 10 =$

5. (6pts)  $8x^2 + 10x - 3 =$

6. (8pts) Compute expressions with fractions by hand.

$$\frac{3}{2} \cdot \frac{14}{9} =$$

$$\frac{7}{10} \div \frac{28}{15} =$$

$$\frac{7}{6} + \frac{2}{9} =$$

$$\frac{19}{24} - \frac{13}{20} =$$

Multiply or divide the rational expressions.

7. (7pts)  $\frac{3x + 12}{x^2 + 7x + 12} \cdot \frac{x^2 - 9}{4x - 12} =$

8. (7pts)  $\frac{x^2 - 2x - 24}{9x + 3} \div \frac{x^2 - 8x + 12}{3x^2 - 14x - 5} =$

Add or subtract the rational expressions.

9. (6pts)  $\frac{x + 1}{x - 5} - \frac{x^2 + 3}{x^2 - 2x - 15} =$

10. (8pts)  $\frac{x + 2}{x^2 - 2x - 35} + \frac{x - 2}{2x^2 - 11x - 21} =$

College Algebra — Joysheet 8  
MAT 140, Fall 2021 — D. Ivanišić

Name: \_\_\_\_\_

*Covers: 3.1, 3.2, 3.3 Show all your work!*

Simplify, so that the answer is in form  $a + bi$ .

1. (4pts)  $(4 + i)^2 + 3i =$

2. (6pts)  $\frac{5 - i}{1 + 3i} =$

3. (4pts) Simplify and justify your answer.

$i^{77} =$

4. (8pts) The number of barrels of wine in the cellar of a winery is described by the function  $S(x) = -x^2 + 24x - 8$ , where  $x$  is the number of days after September 17th.

a) On what dates did the winery have 120 barrels of wine in the cellar?

b) On what date did the number of barrels of wine in the cellar peak?

5. (8pts) Solve the equation:  $x^4 - 2x^2 - 35 = 0$

6. (6pts) Solve by completing the square.

$x^2 + 8x + 13 = 0$

**7.** (12pts) The quadratic function  $f(x) = x^2 + 3x - 18$  is given. Do the following without using the calculator.

- a) Find the  $x$ -intercepts of its graph, if any. Find the  $y$ -intercept.
- b) Find the vertex of the graph.
- c) Sketch the graph of the function.

**8.** (12pts) A right triangle has sides of length 11 and 9 meters (neither is the hypotenuse). We wish to add an equal amount of length to both sides so that we get a right triangle with hypotenuse 16 meters. By how much should the 11- and 9-meter sides be lengthened?



College Algebra — Joysheet 9  
MAT 140, Fall 2021 — D. Ivanišić

Name: \_\_\_\_\_

*Covers: 3.3, 3.4, 3.5 Show all your work!*

1. (4pts) Solve the equation.

$$|5x + 2| = 4$$

2. (12pts) Solve the inequalities. Draw your solution and write it in interval form.

$$|x - 6| \leq 2$$

$$|4x + 3| > 6$$

Solve the equations:

3. (8pts)  $\frac{x}{x-2} + \frac{4}{x+3} = \frac{10}{x^2+x-6}$

4. (8pts)  $\sqrt{46-5x} + 10 = x$

5. (14pts) A ball is launched from height 4 meters upwards with initial velocity 20 meters per second. Its height in meters after  $t$  seconds is given by  $s(t) = -5t^2 + 20t + 4$ .

a) Sketch the graph of the height function.

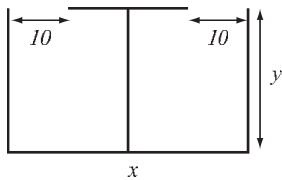
b) When does the ball reach its greatest height, and what is that height?

c) When is the ball at height 20.8 meters?

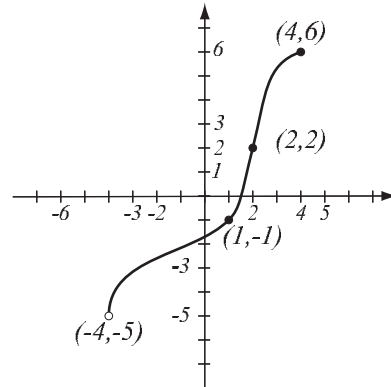
6. (14pts) A business real estate company is planning a building meant to house two stores, each with doors 10 feet wide (see picture). The company has budgeted for total wall length 900 feet and its goal is to maximize the enclosed area.

a) Express the area of the building as a function of one of the sides of the rectangle. What is the domain of this function?

c) Sketch the graph of the area function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the building that has the greatest area and what is the greatest area possible?



1. (6pts) The graph of a function  $f$  is given.
- Is this function one-to-one? Justify.
  - If the function is one-to-one, find the graph of  $f^{-1}$ , labeling the relevant points.



2. (12pts) Let  $f(x) = \frac{x}{x+1}$ . Find the formula for  $f^{-1}$ . Find the ranges of  $f$  and  $f^{-1}$ .

3. (8pts) Evaluate without using the calculator. For each problem, write the question you should ask yourself in order to find the logarithms.

$\log 100 =$

$\log_4 \frac{1}{16} =$

$\log_{16} 2 =$

$\log_{a^2} a^8 =$

4. (4pts) Use the change-of-base formula and your calculator to find  $\log_{13} 9$  with accuracy 6 decimal places. Show how you obtained your number.

5. (12pts) Investigate the effect of increased frequency of compounding: for a deposit of \$5,000 and annual interest rate of 3.48%, calculate the amount in the account after 1 year for the frequencies of compounding below.

- Write the general formula for the amount, replacing the variables by numbers, if known.
- Use the table feature on your calculator to quickly compute amounts after 1 year.
- Does compounding more often make a big difference?

Frequency: every	$n$	Amount after 1 year
year		
quarter		
month		
day		
hour		
second		

6. (3pts) Find the domain of  $f(x) = \ln(7x - 3)$ .

7. (8pts) The distance a falling object travels is given approximately by  $s(t) = 5t^2$ , where  $s$  is in meters and  $t \geq 0$  is in seconds.

- Find the distance a falling object has traveled after 2 seconds and 5 seconds.
- Find a formula for the inverse function and explain what it represents.
- Find how long it takes an object to travel 80 meters and 200 meters.

8. (7pts) Using transformations, draw the graph of  $f(x) = -\ln(x + 7)$ . Explain how you transform the graph of a basic function in order to get the graph of  $f$ . Show at least one point on the graph, and asymptotes to the graph, if any.

1. (5pts) If  $\log_a 7 = u$  and  $\log_a 3 = v$ , express in terms of  $u$  and  $v$ :

$$\log_a 69 = \qquad \qquad \qquad \log_a \frac{3}{7} =$$

2. (11pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$$\log_3 (9x^4y^8) =$$

$$\ln \frac{\sqrt{x^5y^6}}{e^5x^{\frac{3}{2}}y^2} =$$

3. (12pts) Write as a single logarithm. Simplify if possible.

$$2 \log_4(6u^3) + 3 \log_4 v^7 - 2 \log_4(3u^2) =$$

$$\log(x - 7) + 3 \log(x + 3) - 2 \log(x^2 - 4x - 21) =$$

4. (4pts) Simplify.

$$\log_5 5^{3x-1} =$$

$$10^{\log 140} =$$

5. (8pts) Convert equation into other form, logarithmic or exponential.

$$12 = x^3$$

$$\log_3 x = 2$$

$$4^a = 15$$

$$\log_c 6 = \frac{1}{2}$$

6. (6pts) A store bought a refrigerated display case for \$2,000. The value of the case each year is 90% of the value of the year before, so after  $t$  years its value is given by the function  $V(t) = 2000 \cdot 0.9^t$ . When will the value of the case be \$500?

7. (14pts) The town of Risington had 15,000 inhabitants in 2018 and 19,000 in 2021. Assume the population of Risington grows exponentially.

a) Write the function describing the number  $P(t)$  of people in Risington  $t$  years after 2018. Then find the exponential growth rate for this population.

b) Graph the function.

c) According to this model, when will the population reach 25,000?