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| **College Algebra — Joysheet 11**<br>**MAT 140, Fall 2021 — D. Ivanšić** | **Name:** Saul Ocean<br>*Covers: 5.3, 5.4, 5.6 Show all your work!* |

**1.** (5pts) If $\log_a 7 = u$ and $\log_a 3 = v$, express in terms of $u$ and $v$:

$\log_a \cancel{69}\,63 = \log_a(7\cdot 3^2)$

$= \log_a 7 + \log_a 3^2$

$= \log_a 7 + 2\log_a 3$

$= u + 2v$

$\log_a \frac{3}{7} = \log_a 3 - \log_a 7$

$= v - u$

**2.** (11pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$\log_3(9x^4y^8) = \log_3 9 + \log_3 x^4 + \log_3 y^8$

$= 2 + 4\log_3 x + 8\log_3 y$

$\ln \frac{\overbrace{\sqrt{x^5}}^{x^{\frac{5}{2}}}y^6}{e^5x^{\frac{3}{2}}y^2} = \ln(x^{\frac{5}{2}}y^6) - \ln(e^5x^{\frac{3}{2}}y^2) = \ln x^{\frac{5}{2}} + \ln y^6 - (\ln e^5 + \ln x^{\frac{3}{2}} + \ln y^2)$

$= \frac{5}{2}\ln x + 6\ln y - 5 - \frac{3}{2}\ln x - 2\ln y = \ln x + 4\ln y - 5$

$\frac{5}{2}-\frac{3}{2}=1$ $\quad 6-2=4$

**3.** (12pts) Write as a single logarithm. Simplify if possible.

$2\log_4(6u^3) + 3\log_4 v^7 - 2\log_4(3u^2) = \log_4(6u^3)^2 + \log_4(v^7)^3 - \log_4(3u^2)^2$

$= \log_4 \frac{(6u^3)^2(v^7)^3}{(3u^2)^2} = \log_4 \frac{\cancel{36}^{4}\,u^{\cancel{6}\,2}\cdot v^{21}}{\cancel{9}\,\cancel{u^4}} = \log_4(4u^2v^{21})$

$\log(x-7) + 3\log(x+3) - 2\log(x^2-4x-21) = \log(x-7) + \log(x+3)^3 - \log(x^2-4x-21)^2$

$= \log \frac{(x-7)(x+3)^3}{\underbrace{(x^2-4x-21)}_{=(x-7)(x+3)}{}^2} = \log \frac{\cancel{(x-7)}(x+3)^{\cancel{3}\,1}}{(x-7)^{\cancel{2}\,1}\cancel{(x+3)^2}} = \log \frac{x+3}{x-7}$

**4.** (4pts) Simplify.

$\log_5 5^{3x-1} =$ $3x-1$ $\qquad$ $10^{\log 140} =$ $140$

**5.** (8pts) Convert equation into other form, logarithmic or exponential.

$12 = x^3$ $\quad \log_x 12 = 3$ $\qquad$ $\log_3 x = 2$ $\quad 3^2 = x$

$4^a = 15$ $\quad \log_4 15 = a$ $\qquad$ $\log_c 6 = \frac{1}{2}$ $\quad c^{\frac{1}{2}} = 6$

$\sqrt{c} = 6$

**6.** (6pts) A store bought a refrigerated display case for \$2,000. The value of the case each year is 90% of the value of the year before, so after $t$ years its value is given by the function $V(t) = 2000 \cdot 0.9^t$. When will the value of the case be \$500?

$2000 \cdot 0.9^t = 500 \quad | \div 2000$

$0.9^t = \frac{1}{4} \quad | \ln$

$\ln 0.9^t = \ln \frac{1}{4}$

$t \ln 0.9 = \ln \frac{1}{4}$

$t = \frac{\ln \frac{1}{4}}{\ln 0.9} = 13.157627$

In about 13 years.

**7.** (14pts) The town of Risington had 15,000 inhabitants in 2018 and 19,000 in 2021. Assume the population of Risington grows exponentially.
a) Write the function describing the number $P(t)$ of people in Risington $t$ years after 2018. Then find the exponential growth rate for this population.
b) Graph the function.
c) According to this model, when will the population reach 25,000?

a) $P(t) = 15 e^{kt}$

| $t$ | $P(t)$ |
|---|---|
| 0 | 15 |
| 3 | 19 |

$19 = 15 e^{k \cdot 3}$

$\frac{19}{15} = e^{k \cdot 3} \quad | \ln$

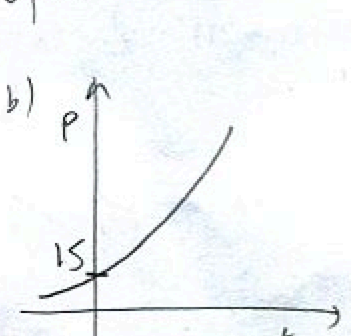
$\ln \frac{19}{15} = \ln e^{k \cdot 3}$

$\ln \frac{19}{15} = 3k$

$k = \frac{\ln \frac{19}{15}}{3} = 0.0787963$

$P(t) = 15 e^{0.0787963t}$

b)




c) Need $t$ so that

$15 e^{0.0787..t} = 25$

$e^{0.0787..t} = \frac{25}{15} \quad | \ln$

$\ln e^{0.0787..t} = \ln \frac{5}{3}$

$0.0787..t = \ln \frac{5}{3}$

$t = \frac{\ln \frac{5}{3}}{0.0787..} = 6.482866$

$2018 + 6.5 = 2024.5$

Mid-2024 pop. should reach 25K