College Algebra — Joysheet 10
MAT 140, Fall 2021 — D. Ivanšić

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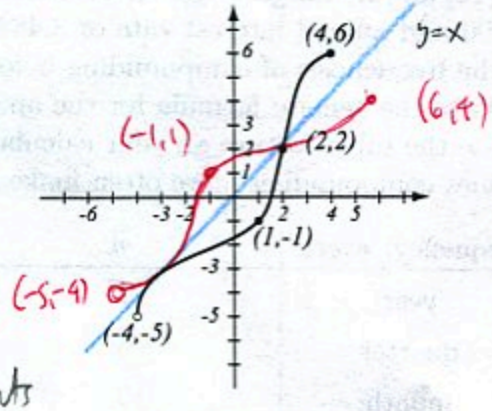
Covers: 5.1–5.3 *Show all your work!*

1. (6pts) The graph of a function f is given.
a) Is this function one-to-one? Justify.
b) If the function is one-to-one, find the graph of f^{-1}, labeling the relevant points.



a) Yes - it passes the horizontal line test

b) Swap x, and y-coordinates of points

2. (12pts) Let $f(x) = \dfrac{x}{x+1}$. Find the formula for f^{-1}. Find the ranges of f and f^{-1}.

$y = \dfrac{x}{x+1}$ $\quad f^{-1}(y) = \dfrac{y}{1-y}$

$y(x+1) = x$

$yx + y = x$

$yx - x = -y$

$x(y-1) = -y$

$x = -\dfrac{y}{y-1} = \dfrac{y}{1-y}$

Range of f = Domain of f^{-1}

Can't have $1-y=0$

$y=1$

$(-\infty, 1) \cup (1, \infty)$

Range of f^{-1} = Domain of f

Can't have $x+1=0$

$x=-1$

$(-\infty, -1) \cup (-1, \infty)$

3. (8pts) Evaluate without using the calculator. For each problem, write the question you should ask yourself in order to find the logarithms.

$\log 100 = 2$ $\quad \log_4 \frac{1}{16} = -2$ $\quad \log_{16} 2 = \frac{1}{4}$ $\quad \log_{a^2} a^8 = 4$

$10^? = 100$ $\quad 4^? = \frac{1}{16} = \frac{1}{4^2} = 4^{-2}$ $\quad 16^? = 2 = \sqrt[4]{16} = 16^{\frac{1}{4}}$ $\quad (a^2)^? = a^8$ $\quad a^{2\cdot?} = a^8$ $\quad ? = 4$

4. (4pts) Use the change-of-base formula and your calculator to find $\log_{13} 9$ with accuracy 6 decimal places. Show how you obtained your number.

$\log_{13} 9 = \dfrac{\ln 9}{\ln 13} = 0.856635$

5. (12pts) Investigate the effect of increased frequency of compounding: for a deposit of \$5,000 and annual interest rate of 3.48%, calculate the amount in the account after 1 year for the frequencies of compounding below.
a) Write the general formula for the amount, replacing the variables by numbers, if known.
b) Use the table feature on your calculator to quickly compute amounts after 1 year.
c) Does compounding more often make a big difference?

Frequency: every	n	Amount after 1 year
year	1	5174
quarter	4	5176.28
month	12	5176.80
day	365	5177.05
hour	365.24	5177.06
second	365·24·3600	5177.06

$A = 5000\left(1 + \frac{0.0348}{n}\right)^{n}$

c) Increasing beyond quarterly compounding makes very little difference.

6. (3pts) Find the domain of $f(x) = \ln(7x - 3)$.

Must have $7x - 3 > 0$

$7x > 3$

$x > \frac{3}{7}$

$(\frac{3}{7}, \infty)$

7. (8pts) The distance a falling object travels is given approximately by $s(t) = 5t^2$, where s is in meters and $t \geq 0$ is in seconds.
a) Find the distance a falling object has traveled after 2 seconds and 5 seconds.
b) Find a formula for the inverse function and explain what it represents.
c) Find how long it takes an object to travel 80 meters and 200 meters.

a) $s(2) = 5 \cdot 2^2 = 20$ m

$s(5) = 5 \cdot 5^2 = 125$ m

b) $s = 5t^2$ $\quad t = \pm\sqrt{\frac{s}{5}}$

$\frac{s}{5} = t^2$ $\quad$ Take + since $t \geq 0$

$t = \sqrt{\frac{s}{5}} = s^{-1}(s)$ gives time an object needs to drop s meters

c) $t = \sqrt{\frac{80}{5}} = \sqrt{16} = 4$

$t = \sqrt{\frac{200}{5}} = \sqrt{40} = 2\sqrt{10} = 6.324555$

8. (7pts) Using transformations, draw the graph of $f(x) = -\ln(x + 7)$. Explain how you transform the graph of a basic function in order to get the graph of f. Show at least one point on the graph, and asymptotes to the graph, if any.