College Algebra — Joysheet 9
MAT 140, Fall 2021 — D. Ivanšić

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Covers: 3.3, 3.4, 3.5 Show all your work!

1. (4pts) Solve the equation.

$|5x+2|=4$

$5x+2=4$ or $5x+2=-4$

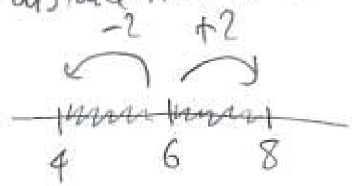
$5x=2$ $\quad$ $5x=-6$

$x=\frac{2}{5}$ $\quad$ $x=-\frac{6}{5}$

2. (12pts) Solve the inequalities. Draw your solution and write it in interval form.

$|x-6|\le 2$

distance from x to $6 \le 2$

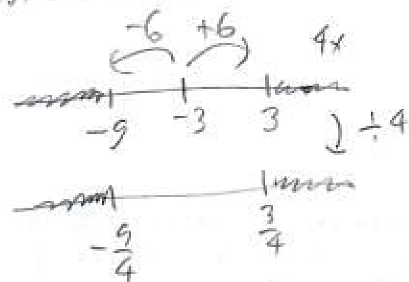


$[4,8]$

$|4x+3|>6$

$|4x-(-3)|>6$

distance from $4x$ to $-3 > 6$



$(-\infty,-\frac{9}{4})\cup(\frac{3}{4},\infty)$

Solve the equations:

3. (8pts) $\dfrac{x}{x-2}+\dfrac{4}{x+3}=\dfrac{10}{x^2+x-6}$ $\quad | \cdot (x+3)(x-2)$

$(x+3)(x-2)$

$\frac{x}{\cancel{x-2}}(x+3)\cancel{(x-2)}+\frac{4}{\cancel{x+3}}\cancel{(x+3)}(x-2)=\frac{10}{\cancel{(x+3)}\cancel{(x-2)}}\cancel{(x+3)}\cancel{(x-2)}$

$x(x+3)+4(x-2)=10$

$x^2+3x+4x-8=10 \quad |-10$

$x^2+7x-18=0$

$(x+9)(x-2)=0$

$x=-9, \cancel{2}$ gives 0 in denominator of original eq.

4. (8pts) $\sqrt{46-5x}+10=x$ $\quad |-10$

$\sqrt{46-5x}=x-10 \quad |^2$

$46-5x=x^2-2\cdot x\cdot 10+10^2 \quad |-46+5x$

$x^2-20x+100-46+5x=0$

$x^2-15x+54=0$

$(x-6)(x-9)=0$

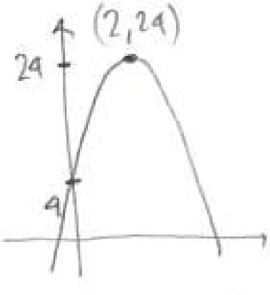
$x=6,9$

Check: $\sqrt{46-5\cdot 6}\overset{?}{=}6-10$ $\quad$ $4\overset{?}{=}-4$ no

$\sqrt{46-5\cdot 9}\overset{?}{=}9-10$ $\quad$ $1\overset{?}{=}-1$ no

5. (14pts) A ball is launched from height 4 meters upwards with initial velocity 20 meters per second. Its height in meters after t seconds is given by $s(t) = -5t^2 + 20t + 4$.
a) Sketch the graph of the height function.
b) When does the ball reach its greatest height, and what is that height?
c) When is the ball at height 20.8 meters?

a)



b) $h = -\frac{b}{2a} = -\frac{20}{2(-5)} = 2$

$k = s(2) = -5 \cdot 2^2 + 20 \cdot 2 + 4$
$= 24$

After 2 seconds ball is at highest point of 24 meters

c) $-5t^2 + 20t + 4 = 20.8 \quad |-20.8$

$-5t^2 + 20t - 16.8 = 0$

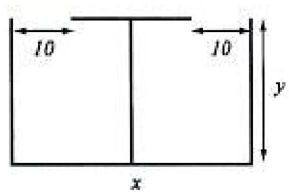
$5t^2 - 20t + 16.8 = 0 \quad 400 - 336 = 64$

$t = \frac{-(-20) \pm \sqrt{(-20)^2 - 4 \cdot 5 \cdot 16.8}}{2 \cdot 5}$

$= \frac{20 \pm 8}{10} = \frac{28}{10}, \frac{12}{10} = 2.8, 1.2$

At $t = 1.2s, 2.8s$ ball is at height 20.8 meters

6. (14pts) A business real estate company is planning a building meant to house two stores, each with doors 10 feet wide (see picture). The company has budgeted for total wall length 900 feet and its goal is to maximize the enclosed area.
a) Express the area of the building as a function of one of the sides of the rectangle. What is the domain of this function?
c) Sketch the graph of the area function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the building that has the greatest area and what is the greatest area possible?



$x + x - 20 + 3y = 900$

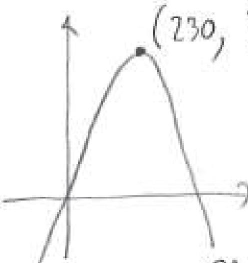
$2x + 3y = 920$

$3y = 920 - 2x$

$y = \frac{920 - 2x}{3} = \frac{920}{3} - \frac{2}{3}x$

$A = xy = x \cdot \left(\frac{920}{3} - \frac{2}{3}x\right)$

$= -\frac{2}{3}x^2 + \frac{920}{3}x$



$h = -\frac{b}{2a} = -\frac{\frac{920}{3}}{2 \cdot \left(-\frac{2}{3}\right)} = \frac{920}{3} \cdot \frac{3}{4} = 230$

$k = -\frac{2}{3}230^2 + \frac{920}{3} \cdot 230 = 35,266.66667$

$\frac{920 - 2 \cdot 230}{3}$

Domain:
must have $x \geq 20$

$y \geq 0$

$\frac{920 - 2x}{3} \geq 0 \quad |\cdot 3$

$920 - 2x \geq 0$

$920 \geq 2x$

$460 \geq x$

Domain: $[20, 460]$

Dimensions: $\boxed{230 \times 153.333333}$

Max area possible: $\boxed{35,266.66667 \text{ ft}^2}$