College Algebra — Joysheet 8
MAT 140, Fall 2021 — D. Ivanšić

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Covers: 3.1, 3.2, 3.3 Show all your work!

Simplify, so that the answer is in form $a+bi$.

1. (4pts) $(4+i)^2+3i =$ $4^2+2\cdot4\cdot i+i^2+3i$

$= 16+8i-1+3i = 15+11i$

2. (6pts) $\dfrac{5-i}{1+3i} =$ $\dfrac{5-i}{1+3i}\cdot\dfrac{1-3i}{1-3i} = \dfrac{5-15i-i+3i^2}{1^2-(3i)^2} = \dfrac{5-16i-3}{1-9i^2} = \dfrac{2-16i}{10}$

$= \dfrac{2(1-8i)}{10} = \dfrac{1-8i}{5}$

3. (4pts) Simplify and justify your answer.

$i^{77} = i^{76}\cdot i = (i^4)^{19}\cdot i = 1\cdot i\cdot i$

4. (8pts) The number of barrels of wine in the cellar of a winery is described by the function $S(x) = -x^2+24x-8$, where x is the number of days after September 17th.

a) On what dates did the winery have 120 barrels of wine in the cellar?

b) On what date did the number of of barrels of wine in the cellar peak?

a) $-x^2+24x-8 = 120$

$-x^2+24x-128 = 0$

$x^2-24x+128 = 0$

$x = \dfrac{-(-24)\pm\sqrt{(-24)^2-4\cdot1\cdot128}}{2\cdot1} = \dfrac{24\pm\sqrt{64}}{2} = \dfrac{24\pm8}{2}$

$x = 16, 8$ On Sep 25th and Oct 3rd

b) $h = -\dfrac{b}{2a} = -\dfrac{24}{2\cdot(-1)} = 12$, On Sep. 29th

5. (8pts) Solve the equation: $x^4-2x^2-35 = 0$

$(x^2)^2-2x^2-35 = 0$

Let $u = x^2$

$u^2-2u-35 = 0$

$(u+5)(u-7) = 0$

$u = -5, 7$

$x^2 = -5$ or $x^2 = 7$

$x = \pm\sqrt{5}i$ $x = \pm\sqrt{7}$

6. (6pts) Solve by completing the square.

$x^2+8x+13 = 0$

$x^2+2\cdot x\cdot4+4^2+13 = 4^2$ $|-13$

$(x+4)^2 = 3$

$x+4 = \pm\sqrt{3}$

$x = -4\pm\sqrt{3}$

7. (12pts) The quadratic function $f(x) = x^2 + 3x - 18$ is given. Do the following without using the calculator.

a) Find the x-intercepts of its graph, if any. Find the y-intercept.

b) Find the vertex of the graph.

c) Sketch the graph of the function.

a) y-int: $f(0) = -18$

x-int: $x^2 + 3x - 18 = 0$

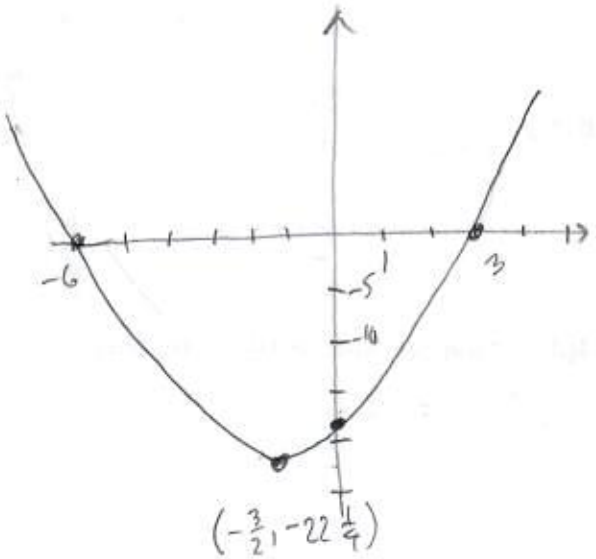
$(x+6)(x-3) = 0$

$x = 3, -6$

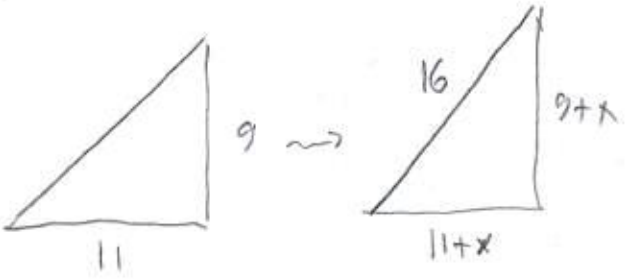
b) $h = -\frac{b}{2a} = -\frac{3}{2 \cdot 1} = -\frac{3}{2}$

$k = f(-\frac{3}{2}) = (-\frac{3}{2})^2 + 3(-\frac{3}{2}) - 18$

$= \frac{9}{4} - \frac{9}{2} - 18 = \frac{9 - 18 - 72}{4} = \frac{-89}{4} = -22\frac{1}{4}$



8. (12pts) A right triangle has sides of length 11 and 9 meters (neither is the hypothenuse). We wish to add an equal amount of length to both sides so that we get a right triangle with hypothenuse 16 meters. By how much should the 11- and 9-meter sides be lengthened?



Pythagorean theorem

$(11+x)^2 + (9+x)^2 = 16^2$

$11^2 + 2 \cdot 11 \cdot x + x^2 + 9^2 + 2 \cdot 9 \cdot x + x^2 = 16^2$

$2x^2 + 40x + 202 = 256 \quad | \div 2$ (22+18, 121+81)

$2x^2 + 40x - 54 = 0$

$x^2 + 20x - 27 = 0$ (400 + 108, 4·127)

$x = \frac{-20 \pm \sqrt{20^2 - 4 \cdot 1 \cdot (-27)}}{2 \cdot 1} = \frac{-20 \pm \sqrt{508}}{2}$

$= \frac{-20 \pm 2\sqrt{127}}{2} = \frac{2(-10 \pm \sqrt{127})}{2} = -10 \pm \sqrt{127}$

$x = -10 \pm \sqrt{127}$

Since $x > 0$, and $-10 - \sqrt{127} < 0$ this cannot be a solution

$x = -10 + \sqrt{127} = 1.269428$