1. (21pts) For the following functions:
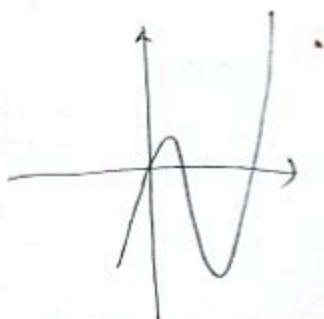a) determine algebraically whether they are odd, even, or neither
b) use the calculator to draw their graphs here and verify your conclusions by stating symmetry.
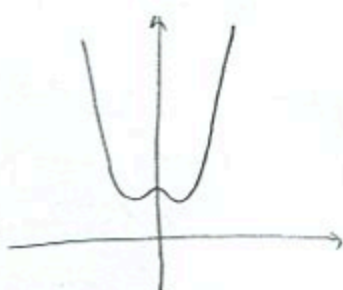
$f(x) = x^3 - 7x^2 + 10x$

$g(x) = x^4 - x^2 + 4$

$h(x) = -x^3 + 7x$

$f(-x) = (-x)^3 - 7(-x)^2 + 10(-x)$
$= -x^3 - 7x^2 - 10x \neq f(x)$
$\neq -f(x)$
neither

$g(-x) = (-x)^4 - (-x)^2 + 4$
$= x^4 - x^2 + 4 = g(x)$
even

$h(-x) = -(-x)^3 + 7(-x)$
$= -(-x^3) - 7x$
$= x^3 - 7x = -h(x)$

no symmetry

symmetric wrt y-axis
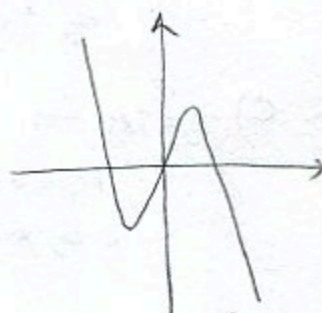
symmetric wrt origin

2. (16pts) Draw the graphs of $f(x) = \sqrt{2x} - 5$ and $g(x) = -2(x+4)^2$ using transformations. Explain how you transform graphs of basic functions in order to get the graphs of f and g. Indicate at least two points on each graph.

$y = \sqrt{x}$

$(0,0)$ $(4,2)$

stretch horiz., factor $= \frac{1}{2}$

$y = \sqrt{2x}$

$(0,0)$ $(2,2)$

shift down 5 $(0,-5)$ $(2,-3)$

$y = x^2$

$(0,0)$ $(2,4)$

shift left 4 $(-4,0)$

$y = (x+4)^2$

$(-2,4)$ $(2,8)$

stretch vertically, factor $= 2$

$y = 2(x+4)^2$

$(-4,0)$ $(-2,8)$

reflect in x-axis
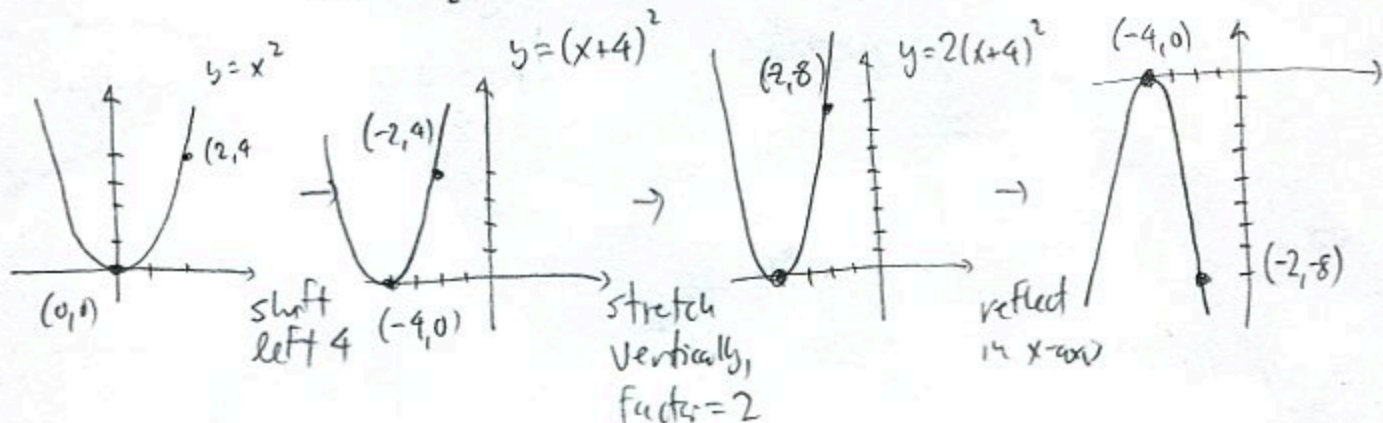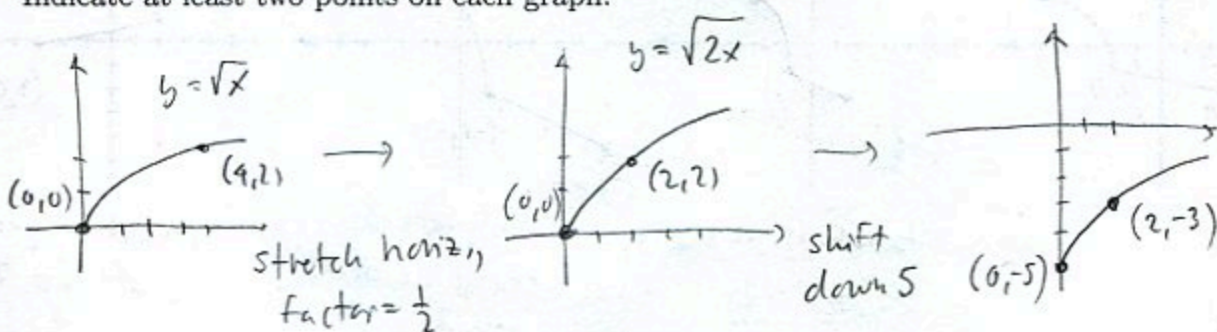
3. (10pts) Write the equation for the function whose graph has the following characteristics:
a) shape of $y = x^3$, shifted down 1 unit
b) shape of $y = \frac{1}{x}$, stretched horizontally by factor 4, then shifted left 2 units
c) shape of $y = |x|$, shifted right 1 unit, then reflected about the y-axis, then stretched vertically by factor 3.

a) $y = x^3 \underset{expr.-1}{\longrightarrow} y = x^3 - 1$

b) $y = \frac{1}{x} \underset{\substack{replace\ x \\ by\ \frac{1}{4}x}}{\longrightarrow} y = \frac{1}{\frac{1}{4}x} \underset{\substack{replace\ x \\ by\ x+2}}{\longrightarrow} y = \frac{1}{\frac{1}{4}(x+2)}$

c) $y = |x| \underset{\substack{replca \\ x\ by\ x-1}}{\longrightarrow} y = |x-1| \underset{\substack{replau \\ x\ by -x}}{\longrightarrow} y = |-x-1| \underset{\substack{3 \cdot entire \\ expr}}{\longrightarrow} y = 3|-x-1|$

4. (13pts) The graph of $f(x)$ is drawn below. On three separate graphs, sketch the graphs of the functions $f(2x)$, $2f(x+3)$ and $f(-x) + 2$ and label all the relevant points.

Graph 1 (original): points $(5,2)$, $(2,-1)$, $(-3,-2)$
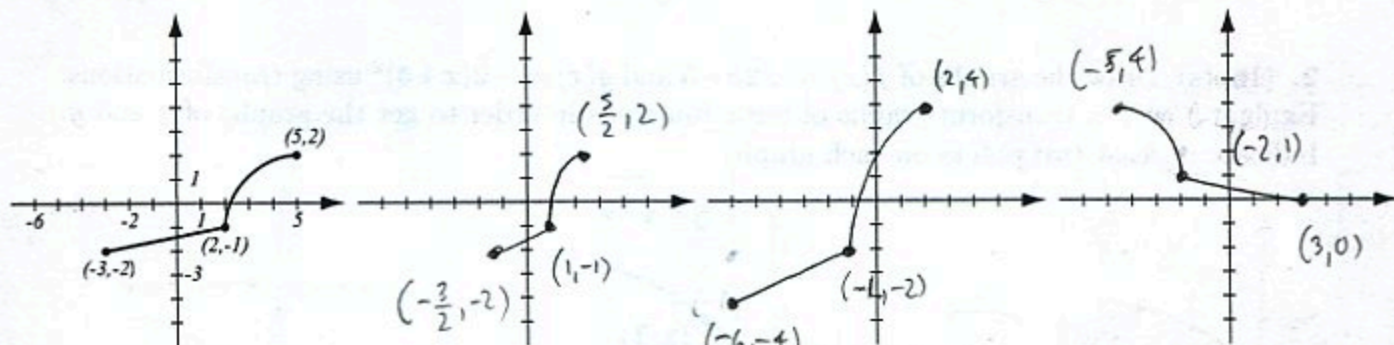
Graph 2: $(\frac{5}{2}, 2)$, $(1,-1)$, $(-\frac{3}{2}, -2)$
stretch horizontally, factor $= \frac{1}{2}$
$x \mapsto \frac{1}{2}x$

Graph 3: $(2,4)$, $(-4,-2)$, $(-6,-4)$
shift left 3
stretch vertically, factor $= 2$
$x \mapsto x-3$
$y \mapsto 2y$

Graph 4: $(-5,4)$, $(-2,1)$, $(3,0)$
reflect in y-axis
shift up 2
$x \mapsto -x$
$y \mapsto y+2$