College Algebra — Joysheet 5
MAT 140, Fall 2021 — D. Ivanšić

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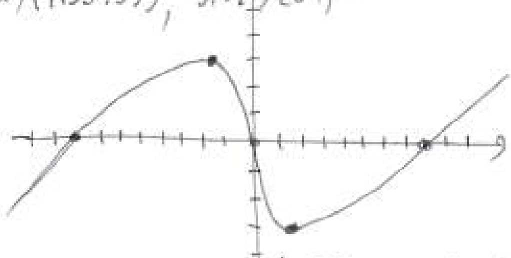
Covers: 2.1, 2.2, 2.3 Show all your work!

1. (10pts) Use your calculator to accurately sketch the graph of the function $f(x) = x - 4\sqrt[3]{x}$. Draw the graph here, indicate units on the axes, and solve the problems below with accuracy 6 decimal points.

a) Find the local maxima and minima for this function.

b) State the intervals where the function is increasing and where it is decreasing.

a) $(-1.539599, 3.079201)$



$(1.539602, -3.079201)$

a) Local max is $3.079201 = f(-1.539599)$

Local min is $-3.079201 = f(1.539602$

b) Increasing on $(-\infty, -1.539599) \cup (1.539602, \infty)$

Decreasing on $(-1.539599, 1.539602$

2. (20pts) Let $f(x) = \dfrac{2}{x^2-4}$, $g(x) = \sqrt{3x-1}$. Find the following (simplify where possible):

$(f+g)(1) = f(1)+g(1) = \dfrac{2}{1^2-4} + \sqrt{3\cdot 1-1} = -\dfrac{2}{3}+\sqrt{2}$

$(fg)(3) = f(3)\cdot g(3) = \dfrac{2}{3^2-4}\cdot\sqrt{3\cdot 3-1}$

$= \dfrac{2}{5}\sqrt{8}$

$\dfrac{g}{f}(x) = \dfrac{g(x)}{f(x)} = \dfrac{\sqrt{3x-1}}{\frac{2}{x^2-4}} = \sqrt{3x-1}\cdot\dfrac{x^2-4}{2}$

$= \dfrac{(x^2-4)\sqrt{3x-1}}{2}$

$(g\circ f)(3) = g(f(3)) = g\left(\dfrac{2}{3^2-4}\right) = g\left(\dfrac{2}{5}\right)$

$= \sqrt{3\cdot\frac{2}{5}-1} = \sqrt{\frac{6}{5}-1} = \sqrt{\frac{1}{5}}$

$(f\circ g)(x) = f(g(x)) = f\left(\sqrt{3x-1}\right) = \dfrac{2}{\sqrt{3x-1}^{\,2}-4}$

$= \dfrac{2}{3x-1-4} = \dfrac{2}{3x-5}$

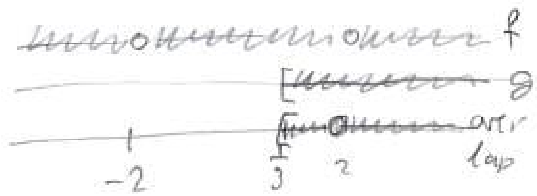
The domain of $(f+g)(x)$ in interval notation

Domain f: Can't have $x^2-4=0$, $x^2=4$, $x=\pm 2$

Domain g: Must have $3x-1 \geq 0$

$3x \geq 1$

$x \geq \frac{1}{3}$



Domain of f+g: $[\frac{1}{3}, 2) \cup (2, \infty)$

3. (8pts) Consider the function $h(x) = \dfrac{7}{x^2+x-6}$ and find **two** different solutions to the following problem: find functions f and g so that $h(x) = f(g(x))$, where neither f nor g are the identity function.

Some examples

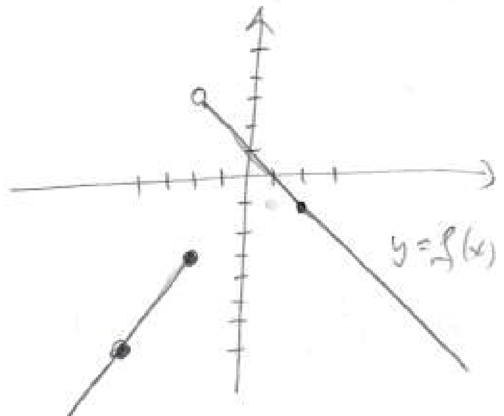
$g(x) = x^2+x-6$	$g(x) = x^2+x$	$g(x) = \dfrac{1}{x^2+x-6}$
$f(x) = \dfrac{7}{x}$	$f(x) = \dfrac{7}{x-6}$	$f(x) = 7x$

4. (8pts) Sketch the graph of the piecewise-defined function:

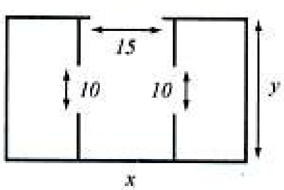
$$f(x) = \begin{cases} 2x+1, & \text{if } x \le -2 \\ 1-x, & \text{if } x > -2. \end{cases}$$

x	2x+1
-2	-3
-4	-7

x	1-x
-2	3
2	-1



5. (14pts) An entrepreneur is designing a simple 3-room store, whose total area is 2500 square feet and which has openings for doors with indicated size. She wishes to minimize construction cost, which is same as minimizing the total length of the walls.
a) Express the total length of the walls of the building as a function of the length of one of the sides x. What is the domain of this function?
b) Graph the function in order to find the minimum. What are the dimensions of the block for which the total length of the walls is minimal? What is the minimal wall length?



a) $x \cdot y = 2500$ so $y = \dfrac{2500}{x}$

$\ell = x + x - 15 + 2y + 2(y-10)$

$= 2x + 4y - 35$

$= 2x + 4 \cdot \dfrac{2500}{x} - 35$

$= 2x + \dfrac{10\,000}{x} - 35$

Domain
Must have:

$x \geq 15 \quad y \geq 10$

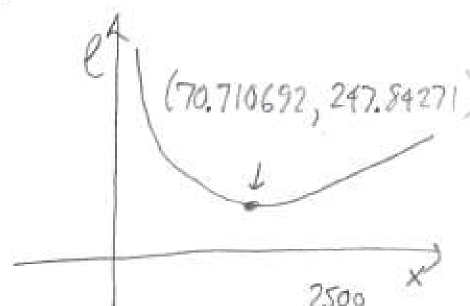
$\dfrac{2500}{x} \geq 10 \quad | \cdot x$

$2500 \geq 10x \quad | \div 10$

$250 \geq x$

$15 \leq x \leq 250$, Domain $= [15, 250]$

b)



$\dfrac{2500}{70.710692}$

Dimensions: $70.710692 \times 35.355332$

Minimal wall length: 247.84271 ft