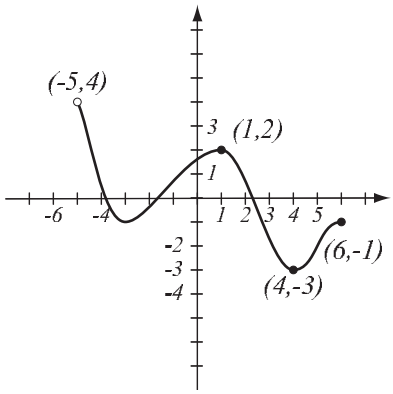
College Algebra — Exam 1
MAT 140C, Fall 2021 — D. Ivanšić

Name: _____

Show all your work!

1. (8pts) Use the graph of the function f at right to answer the following questions.



a) Find: $f(4) =$ $f(1) =$

b) What is the domain of f?

c) What is the range of f?

d) What are the solutions of the equation $f(x) = -2$?

2. (10pts) Use your calculator to accurately sketch the graph of $y = x^3 - 8x^2 + 5x - 3$.

a) Draw the graph on paper and indicate units on the axes.

b) Find all the $x-$ and $y-$intercepts (accuracy: 6 decimal points).

3. (5pts) Draw the line that passes through points $(-1, 1)$ and $(-1, 6)$. Then write the equation of the line.

4. (10pts) Find the equation of the line (in form $y = mx + b$) that is parallel to the line $2x - 4y = 5$ and passes through the point $(4, 1)$. Draw both lines.

5. (8pts) Draw the quadrangle with vertices $A = (4, 0)$, $B = (2, 4)$, $C = (-4, 0)$ and $D = (-2, -3)$. Use slopes to determine if any two of its sides are perpendicular.

6. (9pts) Let $f(x) = x^2 - \sqrt{2x - 7}$. Find the following (simplify where appropriate).

$f(1) =$ $f(8) =$

$f(4u) =$ $f(x + 3) =$

7. (9pts) Find the domains of the functions below and write them using interval notation.

$f(x) = \dfrac{1}{x^2 - 5x - 36}$ $g(x) = \sqrt{2x + 7}$

8. (5pts) Solve and write the solution in interval notation.

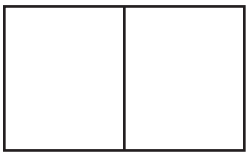
$4 \leq 7 - 2x < 11$

9. (10pts) The diameter of a circle has endpoints $(-2, -3)$ and $(4, 1)$.
a) Find the equation of the circle.
b) Draw the circle in the coordinate plane.

10. (12pts) An electric company offers two plans to pay for electricity usage:
A) \$60 flat fee that includes 200 kWh, then 12 cents per kWh for usage beyond 200 kWh.
B) \$10 flat fee plus 16 cents per kWh.
Assuming a customer always uses at least 200 kWh of electricity, for which amounts of electricity is plan A better?

11. (14pts) Because she was afraid to be late, Fiona rushed to a concert and got there in 2 hours. On the way back, she drove 9mph slower, so it took her a quarter of an hour longer.
a) How fast did Fiona drive to and from the concert?
b) How far did she drive to the concert?

Bonus (10pts) The length of a rectangular field is 40 feet more than the width. A farmer used 470 feet of fencing to enclose the field and divide it into two parts, as in the picture. What are the dimensions of the field?

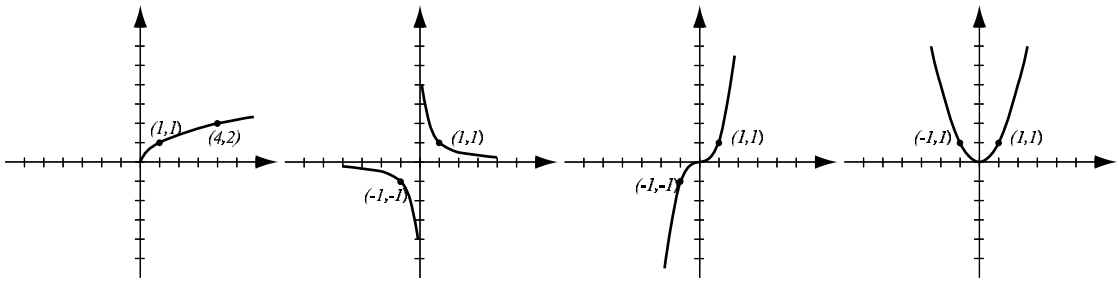


College Algebra — Exam 2
MAT 140C, Fall 2021 — D. Ivanšić

Name: _____

Show all your work!

1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



2. (20pts) Let $f(x) = \sqrt{8-2x}$, $g(x) = \dfrac{x+1}{x^2+4x-12}$.

Find the following (simplify where possible):

$(f+g)(-4) =$

$\dfrac{g}{f}(2) =$

$\dfrac{f}{g}(x) =$

$(f \circ g)(0) =$

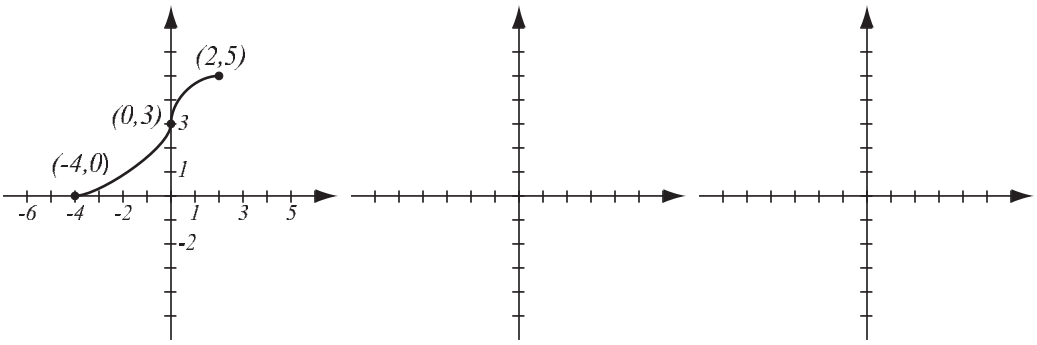
$(g \circ f)(x) =$

The domain of $f - g$ in interval notation

3. (6pts) Consider the function $h(x) = (x^2 + 6)^4$ and find **two** different solutions to the following problem: find functions f and g so that $h(x) = f(g(x))$, where neither f nor g are the identity function.

4. (6pts) Write the equation for the function whose graph has the following characteristics:
a) shape of $y = \sqrt{x}$, stretched vertically by factor 4.
b) shape of $y = x^3$, shifted left 5 units, then reflected over the y axis.

5. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $-f(x-4)$ and $f(2x) - 3$ and label all the relevant points.



6. (8pts) Sketch the graph of the piecewise-defined function:

$$f(x) = \begin{cases} 2x + 1, & \text{if } x \leq 2 \\ x - 4, & \text{if } 2 < x \leq 6 \end{cases}$$

7. (8pts) Find the values of the piecewise-defined function.

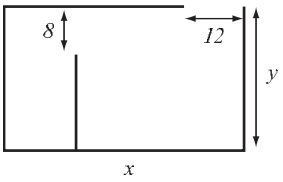
$$f(x) = \begin{cases} 4x - 7, & \text{if } -3 < x \leq 2 \\ \sqrt{x}, & \text{if } 2 < x \leq 10 \\ x^2 - 12x, & \text{if } 10 < x \leq 40 \end{cases}$$

$f(20) =$ $f(9) =$

$f(2) =$ $f(100) =$

8. (20pts) Let $f(x) = x^4 - 8x^2$ (answer with 6 decimal points accuracy).
a) Use your graphing calculator to accurately draw the graph of f (on paper!). Indicate units on the axes.
b) Determine algebraically whether the function is odd, even, or neither.
c) Verify your conclusion from b) by stating symmetry.
d) Find the local maxima and minima for this function. If there is symmetry, use it to reduce the work here.
e) State the intervals where the function is increasing and where it is decreasing.

9. (14pts) A grocery company wishes to build a store that is to have area 6,500 square feet, an entrance door and a door to the rear storage area. To minimize cost, the total length of walls has to be as small as possible.
a) Express the total length of walls of the store as a function of the length of one of the sides x. What is the domain of this function?
b) Graph the function in order to find the minimum. What are the dimensions of the store that has the smallest total wall length?



Bonus. (10pts) Give two functions f and g so that $f(g(x))$ is not equal to $g(f(x))$ (write the formulas for f and g). Verify $f(g(x))$ and $g(f(x))$ are not equal in two ways:
a) Compute $f(g(x))$ and $g(f(x))$ and observe you did not get the same formula.
b) While formulas for $f(g(x))$ and $g(f(x))$ may look different, it may be possible that one can transform one to the other in some difficult way that we cannot see. To rigorously verify that $f(g(x))$ is not equal to $g(f(x))$ give a particular number a so that $f(g(a)) \neq g(f(a))$.

College Algebra — Exam 3
MAT 140C, Fall 2021 — D. Ivanšić

Name: _____

Show all your work!

Simplify, so that the answer is in form $a + bi$.

1. (5pts) $i(4+i)^2 =$

2. (5pts) $\dfrac{3+4i}{7-i} =$

3. (4pts) Simplify and justify your answer.

$i^{102} =$

4. (6pts) Solve the equation by completing the square.

$x^2 - 10x - 12 = 0$

5. (6pts) Solve the inequality. Write the solution in interval form.

$|x - 7| \geq 4$

6. (6pts) Let $P(x)$ be a polynomial of degree 3.
a) Draw a graph of P that has the maximal number of turning points and two x-intercepts.
b) Draw a graph of P that has no turning points.

7. (12pts) The quadratic function $f(x) = x^2 - 2x - 15$ is given. Do the following without using the calculator.
a) Find the x- and y-intercepts of its graph, if any.
b) Find the vertex of the graph.
c) Sketch the graph of the function.

Solve the equations:

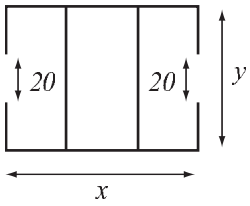
8. (8pts) $\dfrac{x-1}{x-3} = \dfrac{x+5}{x+7} + \dfrac{10-x}{x^2+4x-21}$

9. (8pts) $x + \sqrt{40-3x} = 4$

10. (14pts) The polynomial $f(x) = -\frac{1}{2}(x-4)(x+5)(x+2)^2$ is given.
a) What is the end behavior of the polynomial?
b) List all the zeros and their multiplicities. Find the y-intercept.
c) Use the graphing calculator along with a) and b) to accurately sketch the graph of f (yes, on paper!).
d) Find all the turning points (i.e., local maxima and minima).

11. (12pts) Starting with a $5\,\text{ft} \times 7\,\text{ft}$ rectangle, we increased the width and length by the same amount to get a rectangle with area $50\,\text{ft}^2$. How much was added to the width and length of the 5×7 rectangle?

12. (14pts) A logistics company is building a warehouse whose floorplan is below. It has two entrances of width 20 feet. It has budgeted enough money to build 800 feet of walls, and its goal is to maximize the total area of the warehouse.
a) Express the total area of the warehouse as a function of the length of one of the sides. What is the domain of this function?
b) Graph the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the warehouse that has the biggest possible total area, and what is the biggest possible total area?



Bonus. (10pts) Find the equation of a parabola whose vertex is $(3, -7)$ and whose y-intercept is 5. One way to approach this is to write $y = ax^2 + bx + c$ and find a, b and c based on the information above.

College Algebra — Exam 4
MAT 140C, Fall 2021 — D. Ivanšić

Name: _____

Show all your work!

1. (8pts) Evaluate without using the calculator. For each problem, write the question you should ask yourself in order to find the logarithms.

$\log_2 32 =$ $\log_3 \frac{1}{27} =$ $\log_a \sqrt[5]{a^2} =$ $\log_{a^2} a^6 =$

2. (4pts) Use the change-of-base formula and your calculator to find $\log_7 9$ with accuracy 6 decimal places. Show how you obtained your number.

3. (5pts) If $\log_a 2 = u$ and $\log_a 3 = v$, express in terms of u and v:

$\log_a 6 =$ $\log_a \dfrac{2}{\sqrt{3}} =$

4. (4pts) Simplify.

$\log_6 6^{4x-3} =$ $e^{\ln 3.1} =$

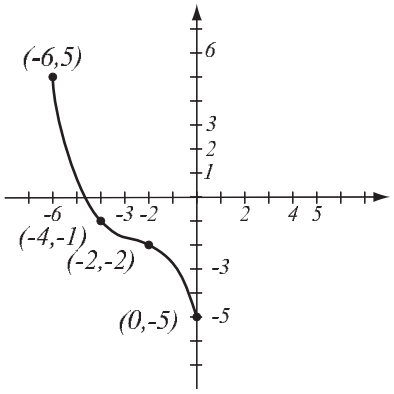
5. (8pts) Convert equation into other form, logarithmic or exponential.

$b = 12^3$ $\log_x 8 = 4$

$e^6 = m$ $\log_6 d = \dfrac{1}{3}$

6. (3pts) Find the domain of the function $f(x) = \ln(8-3x)$ and write it in interval notation.

7. (6pts) The graph of a function f is given.
a) Is this function one-to-one? Justify.
b) If the function is one-to-one, find the graph of f^{-1}, labeling the relevant points, and showing any asymptotes.



8. (9pts) Let $f(x) = \dfrac{x+2}{x}$.
a) Find the formula for f^{-1}.
b) Find the range of f.

9. (6pts) Using transformations, draw the graph of $f(x) = e^{-x} - 2$. Explain how you transform the graph of a basic function in order to get the graph of f. Indicate at least one point on the graph and any asymptotes.

10. (9pts) How much needs to be deposited in an account bearing 8.4% interest, compounded monthly, so that there is \$7,000 in the account after 5 years?

11. (12pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$\log_6 (36x^4y^7) =$

$\log \dfrac{x^3\sqrt{y}}{1000\,x^7y^3} =$

12. (12pts) Write as a single logarithm. Simplify if possible.

$2\log_7(x^3y^2) + 4\log_7(x^{-4}y^3) =$

$6\ln x - 2\ln(x^2+3x) + 4\ln(x+3) =$

13. (14pts) The population of Bloomville was 432,000 in 2015 and 610,000 in 2020. Assume that it has grown according to the formula $P(t) = P_0 e^{kt}$.
a) Find k and write the function that describes the population at time t years since 2015. Graph it on paper.
b) How long will it take until population is 800,000?

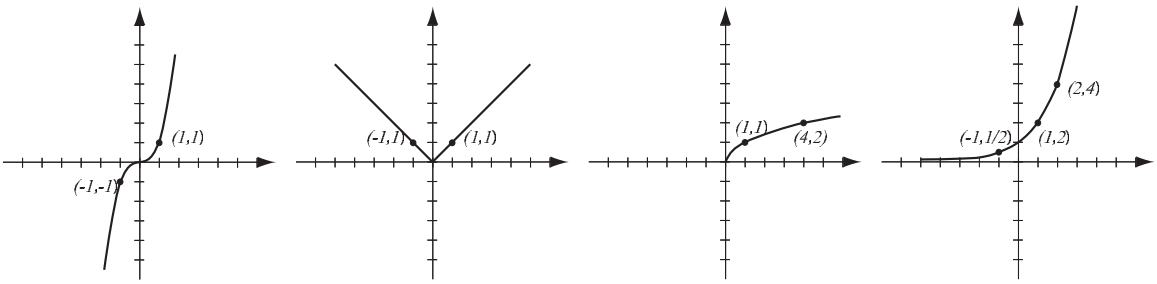
Bonus (10pts) Let $f(x) = \dfrac{e^x - 3}{e^x + 2}$. Find the formula for f^{-1}. *Hint: solve for e^x first.*

College Algebra — Final Exam
MAT 140C, Fall 2021 — D. Ivanšić

Name: _____

Show all your work!

1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



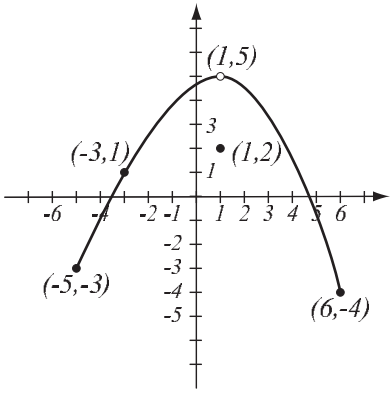
2. (8pts) Use the graph of the function f at right to answer the following questions.

a) Find: $f(-3) =$ $f(1) =$

b) What is the domain of f?

c) What is the range of f?

d) What are the solutions of the equation $f(x) = -3$?



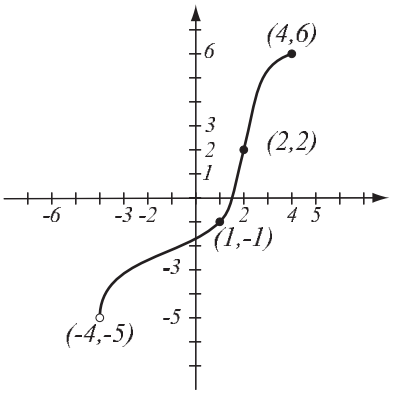
3. (10pts) Find the equation of the line (in form $y = mx + b$) that is perpendicular to the line $2x - 4y = 5$ and passes through the point $(1, -1)$. Draw both lines.

4. (6pts) Solve and write the solution in interval notation.

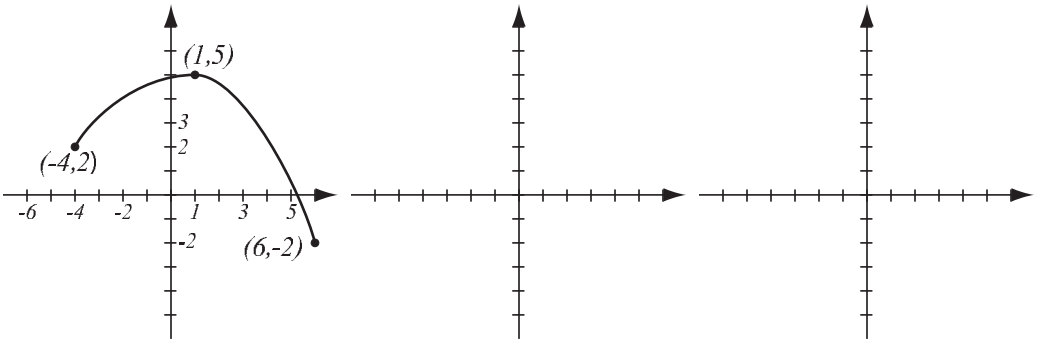
$|x - 8| \geq 2$

5. (4pts) Find the domain of the function $f(x) = \dfrac{\sqrt{x}}{x-5}$ and write it in interval notation.

6. (6pts) The graph of a function f is given.
a) Is this function one-to-one? Justify.
b) If the function is one-to-one, find the graph of f^{-1}, labeling the relevant points.



7. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $-f(x+2)$ and $f(2x)-3$ and label all the relevant points.



8. (12pts) The quadratic function $f(x) = x^2 + 2x - 15$ is given. Do the following without using the calculator.
a) Find the x- and y-intercepts of its graph, if any.
b) Find the vertex of the graph.
c) Sketch the graph of the function.

9. (5pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$\log_4 \left(16x^3\sqrt{y}\right) =$

10. (5pts) Write as a single logarithm. Simplify if possible.

$4\ln(x^3y^2) - \ln(x^3y^6) =$

11. (20pts) The polynomial $P(x) = x^4 - 9x^2$ is given (answer with 6 decimals accuracy).
a) What is the end behavior of the polynomial?
b) Factor the polynomial to find all the zeros and their multiplicities. Find the y-intercept.
c) Determine algebraically whether the function is odd, even, or neither.
d) Use the graphing calculator along with a) and b) to sketch the graph of P (yes, on paper!).
e) Verify your conclusion from c) by stating symmetry.
f) Find all the turning points (i.e., local maxima and minima).

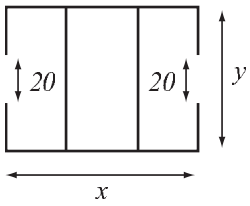
Solve the equations.

12. (8pts) $\dfrac{x-1}{x-3} = \dfrac{x+5}{x+7} + \dfrac{10-x}{x^2+4x-21}$

13. (8pts) $x + \sqrt{40-3x} = 4$

14. (14pts) Because she was afraid to be late, Fiona rushed to a concert and got there in 3 hours. On the way back, she drove 5mph slower, so it took her a quarter of an hour longer.
a) How fast did Fiona drive to and from the concert?
b) How far did she drive to the concert?

15. (14pts) A logistics company is building a warehouse whose floorplan is below. It has two entrances of width 20 feet. It has budgeted enough money to build 1200 feet of walls, and its goal is to maximize the total area of the warehouse.
a) Express the total area of the warehouse as a function of the length of one of the sides. What is the domain of this function?
b) Graph the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the warehouse that has the biggest possible total area, and what is the biggest possible total area?



16. (12pts) The population of Bloomville was 412,000 in 2017 and 498,000 in 2020. Assume that it has grown according to the formula $P(t) = P_0 e^{kt}$.
a) Find k and write the function that describes the population at time t years since 2017. Graph it on paper.
b) Find the predicted population in the year 2025.

Bonus (10pts) Find the equation of a parabola whose vertex is $(3, -7)$ and whose y-intercept is 5. One way to approach this is to write $y = ax^2 + bx + c$ and find a, b and c based on the information above.