College Algebra — Exam 4	Name:	
MAT 140, Spring 2021 — D. Ivanšić		Show all your work!

1. (8pts) Evaluate without using the calculator:

$$\log_3 81 = \log_2 \frac{1}{32} = \log_u \sqrt[7]{a^3} = \log_{\sqrt{a}} a^3 =$$

2. (4pts) Use the change-of-base formula and your calculator to find $\log_5 3$ with accuracy 6 decimal places. Show how you obtained your number.

3. (5pts) If $\log_a 4 = u$ and $\log_a 20 = v$, express in terms of u and v:

$$\log_a 80 = \log_a \sqrt{5} =$$

4. (6pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

 $\log_3\left(9x^2\sqrt[3]{y^8}\right) =$

5. (6pts) Write as a single logarithm. Simplify if possible.

 $3\log_5(x^2y^5) - 4\log_5(x^{-2}y^3) =$

6. (4pts) Simplify. $\ln e^{u-v} =$

 $7^{\log_7 14} =$

7. (6pts) The graph of a function f is given.

a) Is this function one-to-one? Justify.

b) If the function is one-to-one, find the graph of f^{-1} , labeling the relevant points, and showing any asymptotes.



8. (9pts) Let $f(x) = \frac{3x+4}{4x+5}$. a) Find the formula for f^{-1} . b) Find the range of f^{-1} .

9. (6pts) Using transformations, draw the graph of $f(x) = -\log_3(x+2)$. Explain how you transform the graph of a basic function in order to get the graph of f. Indicate at least one point on the graph and any asymptotes.

10. (3pts) Find the domain of the function $f(x) = \ln(5x - 14)$ and write it in interval notation.

11. (9pts) How much needs to be deposited in an account bearing 3.2% interest, compounded quarterly, so that there is \$5,000 in the account after 7 years?

Solve the equations.

12. (6pts)
$$25^{x+1} = \left(\frac{1}{125}\right)^{x-1}$$

13. (8pts) $4^{x-2} = 7^{9x+1}$

14. (8pts) $10^{2x} - 8 \cdot 10^x - 20 = 0$

15. (12pts) According to census data, the population of Kentucky 4,339,367 in 2010 and 4,505,836 in 2020. Assume that it has grown according to the formula $P(t) = P_0 e^{kt}$. a) Find k and write the function that describes the population at time t years since 2010. Graph it on paper.

b) Find the predicted population in the year 2028.

Bonus (10pts) Let $f(x) = \frac{e^x - e^{-x}}{2}$ and $g(x) = x + \sqrt{x^2 + 1}$. Show that f(g(x)) = x, which tells you that g and f are inverses to each other.