## College Algebra - Exam 4 MAT 140, Spring 2021 - D. Ivanšić

1. (8pts) Evaluate without using the calculator:
$\log _{3} 81=$
$\log _{2} \frac{1}{32}=$
$\log _{u} \sqrt[7]{a^{3}}=$
$\log _{\sqrt{a}} a^{3}=$
2. (4pts) Use the change-of-base formula and your calculator to find $\log _{5} 3$ with accuracy 6 decimal places. Show how you obtained your number.
3. (5pts) If $\log _{a} 4=u$ and $\log _{a} 20=v$, express in terms of $u$ and $v$ :
$\log _{a} 80=\quad \quad \log _{a} \sqrt{5}=$
4. (6pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.
$\log _{3}\left(9 x^{2} \sqrt[3]{y^{8}}\right)=$
5. (6pts) Write as a single logarithm. Simplify if possible.
$3 \log _{5}\left(x^{2} y^{5}\right)-4 \log _{5}\left(x^{-2} y^{3}\right)=$
6. (4pts) Simplify.
$\ln e^{u-v}=$

$$
7^{\log _{7} 14}=
$$

7. ( 6 pts ) The graph of a function $f$ is given.
a) Is this function one-to-one? Justify.
b) If the function is one-to-one, find the graph of $f^{-1}$, labeling the relevant points, and showing any asymptotes.

8. (9pts) Let $f(x)=\frac{3 x+4}{4 x+5}$.
a) Find the formula for $f^{-1}$.
b) Find the range of $f^{-1}$.
9. (6pts) Using transformations, draw the graph of $f(x)=-\log _{3}(x+2)$. Explain how you transform the graph of a basic function in order to get the graph of $f$. Indicate at least one point on the graph and any asymptotes.
10. (3pts) Find the domain of the function $f(x)=\ln (5 x-14)$ and write it in interval notation.
11. (9pts) How much needs to be deposited in an account bearing $3.2 \%$ interest, compounded quarterly, so that there is $\$ 5,000$ in the account after 7 years?

Solve the equations.
12. $(6 \mathrm{pts}) 25^{x+1}=\left(\frac{1}{125}\right)^{x-1}$
13. $(8 \mathrm{pts}) 4^{x-2}=7^{9 x+1}$
14. (8pts) $10^{2 x}-8 \cdot 10^{x}-20=0$
15. (12pts) According to census data, the population of Kentucky 4,339,367 in 2010 and $4,505,836$ in 2020. Assume that it has grown according to the formula $P(t)=P_{0} e^{k t}$.
a) Find $k$ and write the function that describes the population at time $t$ years since 2010. Graph it on paper.
b) Find the predicted population in the year 2028.

Bonus (10pts) Let $f(x)=\frac{e^{x}-e^{-x}}{2}$ and $g(x)=x+\sqrt{x^{2}+1}$. Show that $f(g(x))=x$, which tells you that $g$ and $f$ are inverses to each other.

