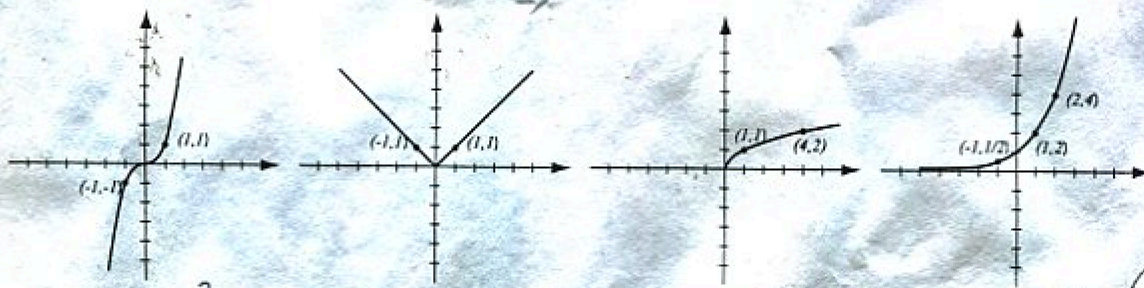
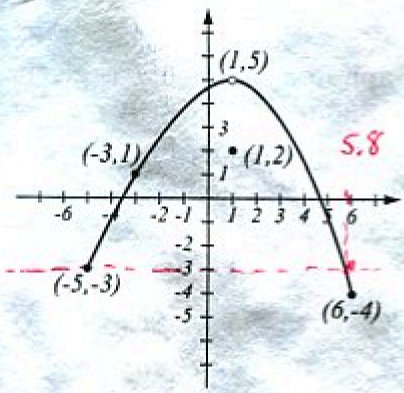
College Algebra — Final Exam **MAT 140C, Fall 2021 — D. Ivanšić**	**Name:** Saul Ocean *Show all your work!*

1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



$y = x^3$ $\quad$ $y = |x|$ $\quad$ $y = \sqrt{x}$ $\quad$ $y = 2^x$ (or $y = b^x$, in general)

2. (8pts) Use the graph of the function f at right to answer the following questions.



a) Find: $f(-3) =$ 1 $\quad f(1) =$ 2

b) What is the domain of f? $[-5, 6]$

c) What is the range of f? $[-4, 5)$

d) What are the solutions of the equation $f(x) = -3$?

$x = -5,\ 5.8$

3. (10pts) Find the equation of the line (in form $y = mx + b$) that is perpendicular to the line $2x - 4y = 5$ and passes through the point $(1, -1)$. Draw both lines.

$2x - 4y = 5$

$4y = 2x - 5 \quad] \div 4$

$y = \frac{1}{2}x - \frac{5}{4}$

slope of this line $= \frac{1}{2}$

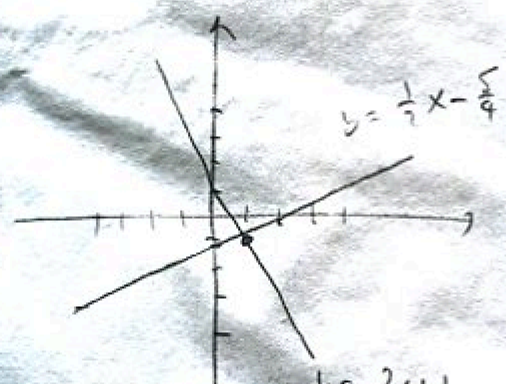
slope of perpendicular line $= -\frac{1}{\frac{1}{2}} = -2$

Eq. of perp. line

$y - (-1) = -2(x - 1)$

$y + 1 = -2x + 2$

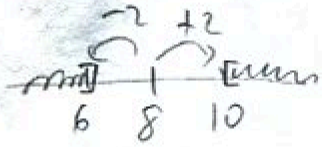
$y = -2x + 1$



4. (6pts) Solve and write the solution in interval notation.

$|x - 8| \geq 2$

distance from x to $8 \geq 2$



$(-\infty, 6) \cup (10, \infty)$

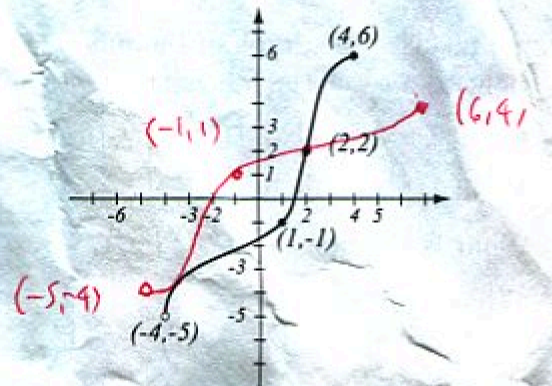
5. (4pts) Find the domain of the function $f(x) = \dfrac{\sqrt{x}}{x-5}$ and write it in interval notation.

Must have: $x \geq 0$

Can't have
$x - 5 = 0$
$x = 5$

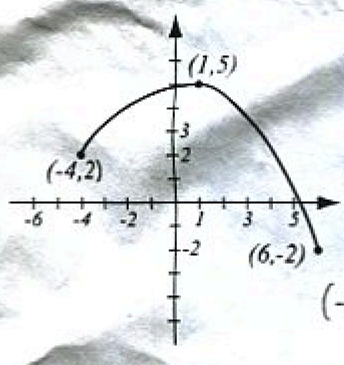
Domain: $[0,5) \cup (5,\infty)$

6. (6pts) The graph of a function f is given.
a) Is this function one-to-one? Justify.
b) If the function is one-to-one, find the graph of f^{-1}, labeling the relevant points.

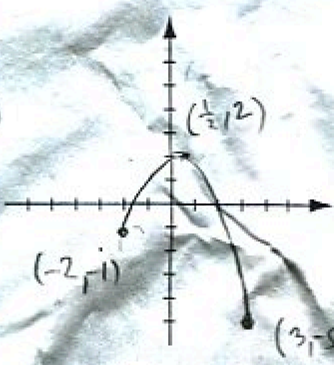


a) Yes, it passes the horizontal line test

7. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $-f(x+2)$ and $f(2x)-3$ and label all the relevant points.



shift left 2
reflect in x-axis



shift down 3
stretch horizontally, factor = 1/2

8. (12pts) The quadratic function $f(x) = x^2 + 2x - 15$ is given. Do the following without using the calculator.

a) Find the x- and y-intercepts of its graph, if any.

b) Find the vertex of the graph.

c) Sketch the graph of the function.

x-int:

a) $x^2 + 2x - 15 = 0$

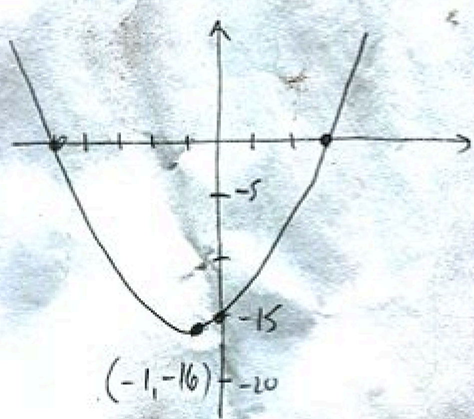
$(x+5)(x-3) = 0$

$x = -5, 3$

y-int: $f(0) = -15$

b) $h = -\frac{b}{2a} = -\frac{2}{2 \cdot 1} = -1$

$k = f(-1) = 1 - 2 - 15 = -16$



9. (5pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$\log_4 (16x^3 \sqrt{y}) = \log_4 16 + \log_4 x^3 + \log_4 y^{\frac{1}{2}}$

$= y^{\frac{1}{2}}$

$= 2 + 3\log_4 x + \frac{1}{2} \log_4 y$

10. (5pts) Write as a single logarithm. Simplify if possible.

$4\ln(x^3y^2) - \ln(x^3y^6) = \ln (x^3y^2)^4 - \ln(x^3y^6)$

$= \ln \frac{(x^3y^2)^4}{x^3y^6} = \ln \frac{x^{12}y^8}{x^3y^6} = \ln(x^9y^2)$

11. (20pts) The polynomial $P(x) = x^4 - 9x^2$ is given (answer with 6 decimals accuracy).
a) What is the end behavior of the polynomial?
b) Factor the polynomial to find all the zeros and their multiplicities. Find the y-intercept.
c) Determine algebraically whether the function is odd, even, or neither.
d) Use the graphing calculator along with a) and b) to sketch the graph of P (yes, on paper!).
e) Verify your conclusion from c) by stating symmetry.
f) Find all the turning points (i.e., local maxima and minima).

a) like x^4 ∪

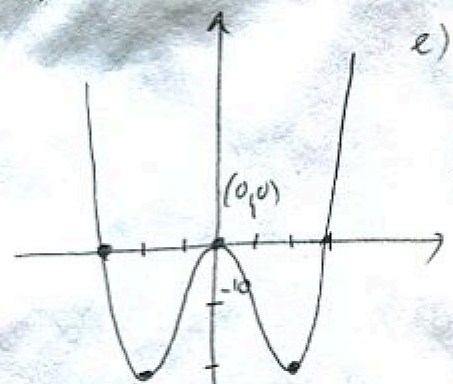
b) $x^4 - 9x^2 = x^2(x^2-9) = x^2(x-3)(x+3)$

zeros	0	3	-3
mult	2	1	1

y-int: $P(0)=0$

c) $P(-x) = (-x)^4 - 9(-x)^2$
$= x^4 - 9x^2$
even function

d)



$(-2.121319, -20.25)$ $(2.121319, -20.25)$

e) symmetric wrt y-axis

f) Local max is $0 = f(0)$
Local min are $-20.25 = f(-2.121319)$
and $-20.25 = f(2.121319)$

Solve the equations.

12. (8pts) $\dfrac{x-1}{x-3} = \dfrac{x+5}{x+7} + \dfrac{10-x}{x^2+4x-21}$ $\Big|\cdot(x+7)(x-3)$

$(x+7)(x-3)$

$\dfrac{x-1}{\cancel{x-3}}(x+7)\cancel{(x-3)} = \dfrac{x+5}{\cancel{x+7}}\cancel{(x+7)}(x-3) + \dfrac{10-x}{\cancel{(x+7)(x-3)}}\cancel{(x+7)(x-3)}$

$(x-1)(x+7) = (x+5)(x-3) + 10 - x$

$x^2+6x-7 = x^2+2x-15+10-x \quad |-x^2$

$6x-7 = x-5 \quad |-x+7$

$5x = 2$

$x = \frac{2}{5}$ ✓ does not give 0 in any denom, so is a solution

13. (8pts) $x + \sqrt{40-3x} = 4$

$\sqrt{40-3x} = 4-x \quad |^2$

$40-3x = 4^2 - 2\cdot 4\cdot x + x^2$

$x^2 - 8x + 16 = 40-3x \quad |+3x-40$

$x^2-5x-24=0$

$(x-8)(x+3)=0$

$x = 8, \boxed{-3}$ ← only solution

Check: $8+\sqrt{40-24} = 4$?
$8+\sqrt{16} = 4$ no
$-3+\sqrt{40+9} = 4$
$-3+7 = 4$ yes

14. (14pts) Because she was afraid to be late, Fiona rushed to a concert and got there in 3 hours. On the way back, she drove 5mph slower, so it took her a quarter of an hour longer.
a) How fast did Fiona drive to and from the concert?
b) How far did she drive to the concert?

$d, r, 3 \text{ hrs} \rightarrow$

$\leftarrow d, r-5, 3.25 \text{ hrs}$

are equal $\begin{cases} d = r \cdot 3 \\ d = (r-5)\,3.25 \end{cases}$

$3r = 3.25(r-5)$

$3r = 3.25r - 16.25 \quad | -3r + 16.25$

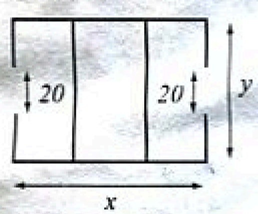
$0.25r = 16.25 \quad | \div 0.25$

$r = 65$ mph

a) To concert: 65 mph
From concert: 60 mph

b) $65 \cdot 3 = 195$ miles

15. (14pts) A logistics company is building a warehouse whose floorplan is below. It has two entrances of width 20 feet. It has budgeted enough money to build 1200 feet of walls, and its goal is to maximize the total area of the warehouse.
a) Express the total area of the warehouse as a function of the length of one of the sides. What is the domain of this function?
b) Graph the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the warehouse that has the biggest possible total area, and what is the biggest possible total area?



$A = xy = (620-2y)y = -2y^2 + 620y$

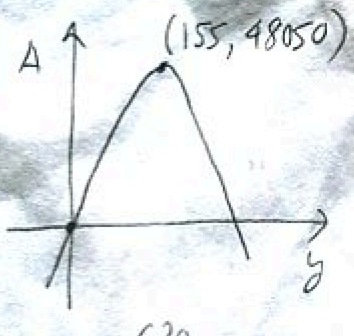
$2x + 2y + 2(y-20) = 1200$

$2x + 4y - 40 = 1200$

$2x + 4y = 1240$

$2x = 1240 - 4y \quad | \div 2$

$x = 620 - 2y$



$h = -\frac{b}{2a} = -\frac{620}{2(-2)} = 155$

$k = (620 - 2 \cdot 155) \cdot 155 = 48{,}050$

Domain:
Must have
$y \geq 20$
$x > 0$
$620 - 2y > 0$
$2y < 620$
$y < 310$

Domain: $[20, 310)$

$x = 620 - 2 \cdot 155$

Dimensions: 310×155

Max area is $48{,}050 \text{ ft}^2$

16. (12pts) The population of Bloomville was 412,000 in 2017 and 498,000 in 2020. Assume that it has grown according to the formula $P(t) = P_0 e^{kt}$.
a) Find k and write the function that describes the population at time t years since 2017. Graph it on paper.
b) Find the predicted population in the year 2025.

a) $P(t) = 412 e^{kt}$ (in thousands)

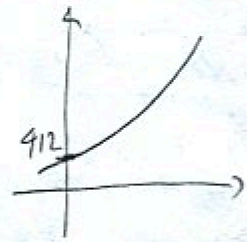
$498 = P(3) = 412 e^{k \cdot 3}$

$\frac{498}{412} = e^{k \cdot 3} \quad | \ln$

$\ln \frac{498}{412} = k \cdot 3$

$k = \frac{\ln \frac{498}{412}}{3} = 0.0631922$

$P(t) = 412 e^{0.0631922t}$



b) 2025 is 8 years after 2017

$P(8) = 412 e^{0.0631922 \cdot 8}$

$= 683.095$ thousand

$= 683,095$ people

Bonus (10pts) Find the equation of a parabola whose vertex is $(3, -7)$ and whose y-intercept is 5. One way to approach this is to write $y = ax^2 + bx + c$ and find a, b and c based on the information above.

$(0,5)$ is on curve, so

$5 = a \cdot 0^2 + b \cdot 0 + c$

so $c = 5$

$(3,-7)$ is on curve so

$-7 = a \cdot 3^2 + b \cdot 3 + 5$

$-7 = 9a + 3b + 5$

Also, since $(3,7)$ is the vertex,

$3 = -\frac{b}{2a}$, so $b = -6a$

Putting $b = -6a$ in first eq., we get,

$-7 = 9a + 3 \cdot (-6a) + 5$

$-12 = -9a$

$a = \frac{-12}{-9} = \frac{4}{3}$, $b = -6 \cdot \frac{4}{3} = -8$

$y = \frac{4}{3}x^2 - 8x + 5$