**College Algebra — Exam 4**
**MAT 140C, Fall 2021 — D. Ivanšić**

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*Show all your work!*

**1.** (8pts) Evaluate without using the calculator. For each problem, write the question you should ask yourself in order to find the logarithms.

$\log_2 32 = 5$ $\quad 2^? = 32$

$\log_3 \frac{1}{27} = -3$ $\quad 3^? = \frac{1}{27} = \frac{1}{3^3} = 3^{-3}$

$\log_a \sqrt[5]{a^2} = \frac{2}{5}$ $\quad a^? = \sqrt[5]{a^2} = a^{\frac{2}{5}}$

$\log_{a^2} a^6 = 3$ $\quad (a^2)^? = a^6$

**2.** (4pts) Use the change-of-base formula and your calculator to find $\log_7 9$ with accuracy 6 decimal places. Show how you obtained your number.

$\log_7 9 = \frac{\ln 9}{\ln 7} \approx 1.12915$

**3.** (5pts) If $\log_a 2 = u$ and $\log_a 3 = v$, express in terms of $u$ and $v$:

$\log_a 6 = \log_a (2 \cdot 3)$
$= \log_a 2 + \log_a 3$
$= u + v$

$\log_a \frac{2}{\sqrt{3}} = \log_a 2 - \log_a 3^{\frac{1}{2}}$
$= \log_a 2 - \frac{1}{2} \log_a 3$
$= u - \frac{1}{2} v$

**4.** (4pts) Simplify.

$\log_6 6^{4x-3} = 4x - 3$

$e^{\ln 3.1} = 3.1$

**5.** (8pts) Convert equation into other form, logarithmic or exponential.

$b = 12^3$ $\quad \log_{12} b = 3$

$\log_x 8 = 4$ $\quad x^4 = 8$

$e^6 = m$ $\quad \ln m = 6$

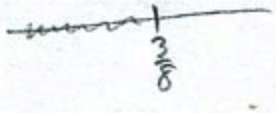
$\log_6 d = \frac{1}{3}$ $\quad 6^{\frac{1}{3}} = d$

**6.** (3pts) Find the domain of the function $f(x) = \ln(8 - 3x)$ and write it in interval notation.

Must have: $8 - 3x > 0$

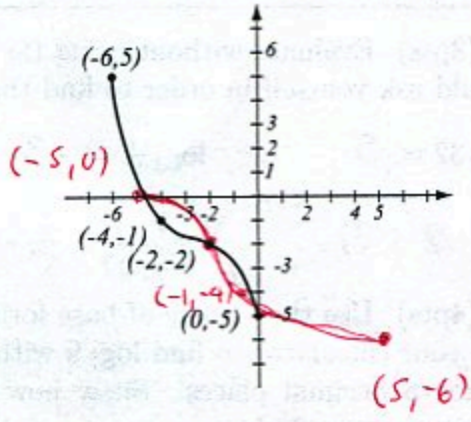
$8 > 3x$

$\frac{8}{3} > x$




$(-\infty, \frac{3}{8})$

**7.** (6pts) The graph of a function $f$ is given.
a) Is this function one-to-one? Justify.
b) If the function is one-to-one, find the graph of $f^{-1}$, labeling the relevant points, and showing any asymptotes.




a) Yes, passes horizontal line test

**8.** (9pts) Let $f(x) = \dfrac{x+2}{x}$.
a) Find the formula for $f^{-1}$.
b) Find the range of $f$.

$y = \dfrac{x+2}{x}$

$yx = x+2$

$yx - x = 2$

$x(y-1) = 2$

$x = \dfrac{2}{y-1}$

$f^{-1}(y) = \dfrac{2}{y-1}$

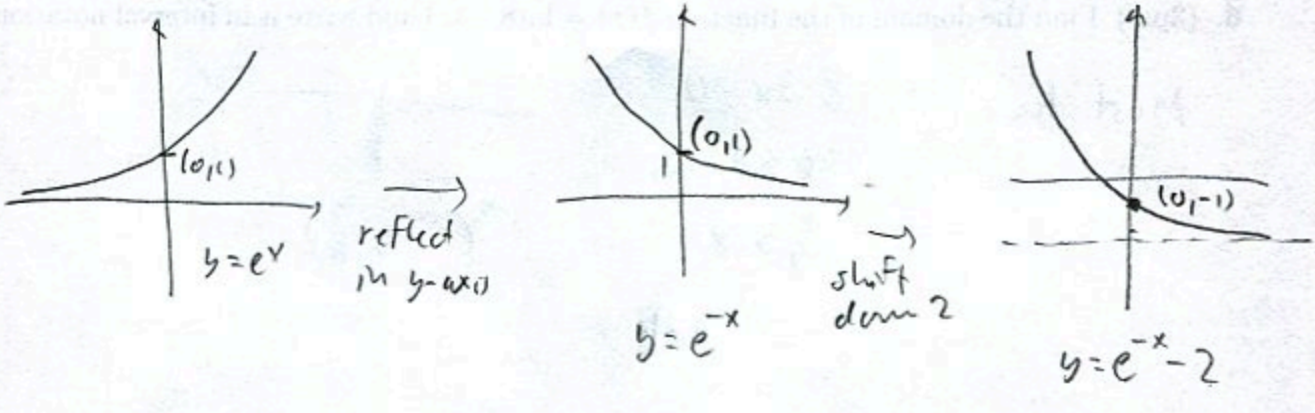
Range of $f$ = domain of $f^{-1}$

Can't have $y-1=0$

$y=1$

Range of $f$ = $(-\infty, 1) \cup (1, \infty)$

**9.** (6pts) Using transformations, draw the graph of $f(x) = e^{-x} - 2$. Explain how you transform the graph of a basic function in order to get the graph of $f$. Indicate at least one point on the graph and any asymptotes.



**10.** (9pts) How much needs to be deposited in an account bearing 8.4% interest, compounded monthly, so that there is \$7,000 in the account after 5 years?

$A = P(1+\frac{r}{n})^{nt}$

$7000 = P\left(1+\frac{0.084}{12}\right)^{12\cdot 5}$

$7000 = P \cdot 1.5197..$

$P = \frac{7000}{1.5197..} = 4606.06$

**11.** (12pts) Write as a sum and/or difference of logarithms. Express powers as factors. Simplify if possible.

$\log_6(36x^4y^7) = \log_6 36 + \log_6 x^4 + \log_6 y^7 = 2 + 4\log_6 x + 7\log_6 y$

$\log \frac{x^3\sqrt{y}}{1000\,x^7y^3} = \log x^3 + \log y^{\frac{1}{2}} - \log 1000 - \log x^7 - \log y^3$

$= 3\log x + \frac{1}{2}\log y - 3 - 7\log x - 3\log y$

$= -4\log x - \frac{5}{2}\log y - 3$

**12.** (12pts) Write as a single logarithm. Simplify if possible.

$2\log_7(x^3y^2) + 4\log_7(x^{-4}y^3) = \log_7 (x^3y^2)^2 + \log_7 (x^{-4}y^3)^4$

$= \log_7\left((x^3y^2)^2 (x^{-4}y^3)^4\right) = \log_7\left(x^6y^4 \cdot x^{-16}y^{12}\right)$

$= \log_7(x^{-10}y^{16}) = \log_7 \frac{y^{16}}{x^{10}}$

$6\ln x - 2\ln(x^2+3x) + 4\ln(x+3) = \ln x^6 - \ln(x^2+3x)^2 + \ln(x+3)^4$

$= \ln \frac{x^6\cdot(x+3)^4}{(x^2+3x)^2} = \ln \frac{x^6(x+3)^4}{(x(x+3))^2} = \ln\left(x^4(x+3)^2\right)$

~~$x^2(x+3)^2$~~

**13.** (14pts) The population of Bloomville was 432,000 in 2015 and 610,000 in 2020. Assume that it has grown according to the formula $P(t) = P_0 e^{kt}$.
a) Find $k$ and write the function that describes the population at time $t$ years since 2015. Graph it on paper.
b) How long will it take until population is 800,000?

a) $P(t) = 432e^{kt}$

| $t$ | $P(t)$ |
|---|---|
| 0 | 432 |
| 5 | 610 |

$610 = 432e^{k\cdot 5}$

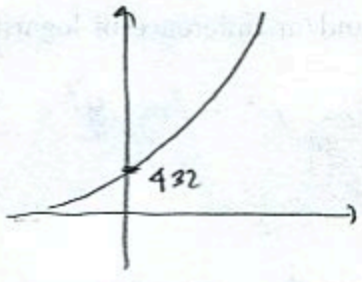
$\frac{610}{432} = e^{k\cdot 5}$ | ln

$\ln\frac{610}{432} = \ln e^{k\cdot 5}$

$\ln\frac{610}{432} = k\cdot 5$

$k = \frac{\ln\frac{610}{432}}{5} = 0.0690067$

$P(t) = 432e^{0.0690067t}$




b) Need $t$ so that

$P(t) = 800$

$432e^{0.069..t} = 800$

$e^{0.069..t} = \frac{800}{432}$ | ln

$0.069..t = \ln\frac{800}{432}$

$t = \frac{\ln\frac{800}{432}}{0.0690067}$

$= 8.92937$, about 9 years

Population is 800,000 in about 2029.

**Bonus** (10pts) Let $f(x) = \frac{e^x - 3}{e^x + 2}$. Find the formula for $f^{-1}$. *Hint: solve for $e^x$ first.*

$y = \frac{e^x - 3}{e^x + 2}$

$y(e^x + 2) = e^x - 3$

$ye^x + 2y = e^x - 3$

$ye^x - e^x = -2y - 3$

$e^x(y-1) = -2y - 3$

$e^x = \frac{-2y-3}{y-1} =$

$e^x = \frac{2y+3}{1-y}$ | ln

$\ln e^x = \ln\frac{2y+3}{1-y}$

$x = \ln\frac{2y+3}{1-y}$