College Algebra — Exam 3
MAT 140C, Fall 2021 — D. Ivanšić

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Show all your work!

Simplify, so that the answer is in form $a + bi$.

1. (5pts) $i(4+i)^2 = i(4^2 + 2\cdot 4\cdot i + i^2) = i(16+8i-1)$

$= i(15+8i) = 15i + 8i^2 = -8 + 15i$

2. (5pts) $\dfrac{3+4i}{7-i} = \dfrac{3+4i}{7-i}\cdot\dfrac{7+i}{7+i} = \dfrac{21+3i+28i+4i^2}{7^2-i^2} = \dfrac{21+31i-4}{49-(-1)}$

$= \dfrac{17+31i}{50}$

3. (4pts) Simplify and justify your answer.

$i^{102} = i^{100}\cdot i^2 = \underbrace{(i^4)}_{=1}{}^{25}\cdot i^2 = i^2 = -1$

4. (6pts) Solve the equation by completing the square.

$x^2 - 10x - 12 = 0 \quad | + 5^2$

$x^2 - 2\cdot x\cdot 5 + 5^2 - 12 = 5^2 \quad | +12$

$(x-5)^2 = 25+12$

$(x-5)^2 = 37$

$x - 5 = \pm\sqrt{37}$

$x = 5 \pm \sqrt{37}$

5. (6pts) Solve the inequality. Write the solution in interval form.

$|x-7| \geq 4$

distance from x to $7 \geq 4$

$(-\infty, 3] \cup [11, \infty)$

$-4 \quad +4$

$3 \quad 7 \quad 11$

6. (6pts) Let $P(x)$ be a polynomial of degree 3.
a) Draw a graph of P that has the maximal number of turning points and two x-intercepts.
b) Draw a graph of P that has no turning points.

a) 2 turning points
2 x-int.

b) easy: x^3

No turning points

7. (12pts) The quadratic function $f(x) = x^2 - 2x - 15$ is given. Do the following without using the calculator.

a) Find the x- and y-intercepts of its graph, if any.

b) Find the vertex of the graph.

c) Sketch the graph of the function.

a) y-int: $f(0) = -15$

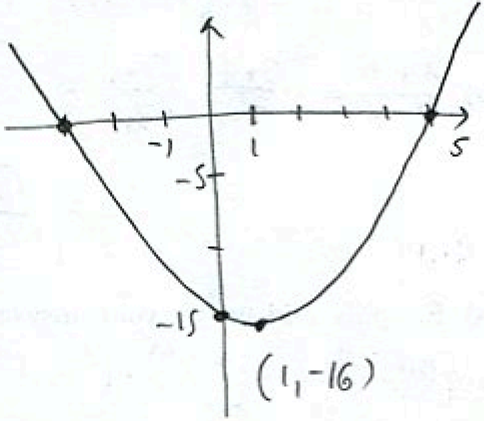
x-int: $x^2 - 2x - 15 = 0$

$(x-5)(x+3) = 0$

$x = 5, -3$

b) $h = -\frac{b}{2a} = -\frac{-2}{2 \cdot 1} = \frac{2}{2} = 1$

$k = f(1) = 1^2 - 2 \cdot 1 - 15 = -16$



Solve the equations:

8. (8pts) $\frac{x-1}{x-3} = \frac{x+5}{x+7} + \frac{10-x}{\cancel{\cdots - 15}} \;\Big|\cdot(x-3)(x+7)$

$(x^2+4x-21)$

$(x-3)(x+7)$

$\frac{x-1}{\cancel{x-3}}(\cancel{x-3})(x+7) = \frac{x+5}{\cancel{x+7}}(x-3)(\cancel{x+7}) + \frac{10-x}{(\cancel{x-3})(\cancel{x+7})}(\cancel{x-3})(\cancel{x+7})$

$(x-1)(x+7) = (x+5)(x-3) + 10 - x$

$\cancel{x^2} + 6x - 7 = \cancel{x^2} + 2x - 15 + 10 - x \quad \Big| -x^2$

$6x - 7 = x - 5 \quad \Big| +7-x$

$5x = 2$

$x = \frac{2}{5}$ ← OK, since does not give 0 in denom.

9. (8pts) $x + \sqrt{40 - 3x} = 4$

$\sqrt{40-3x} = 4 - x \quad \Big|^2$

$40 - 3x = 4^2 - 2 \cdot 4 \cdot x + x^2$

$40 - 3x = 16 - 8x + x^2 \quad \Big| +3x - 40$

$x^2 - 5x - 24 = 0$

$(x+3)(x-8) = 0$

$x = -3, 8$

$\boxed{x = -3 \text{ only sol.}}$

Check: $-3 + \sqrt{40 - 3(-3)} \stackrel{?}{=} 4$

$-3 + \sqrt{49} \stackrel{?}{=} 4$ yes

$8 + \sqrt{40 - 3 \cdot 8} \stackrel{?}{=} 4$

$8 + \sqrt{16} \stackrel{?}{=} 4$ no

10. (14pts) The polynomial $f(x) = -\frac{1}{2}(x-4)(x+5)(x+2)^2$ is given.
a) What is the end behavior of the polynomial?
b) List all the zeros and their multiplicities. Find the y-intercept.
c) Use the graphing calculator along with a) and b) to accurately sketch the graph of f (yes, on paper!).
d) Find all the turning points (i.e., local maxima and minima).

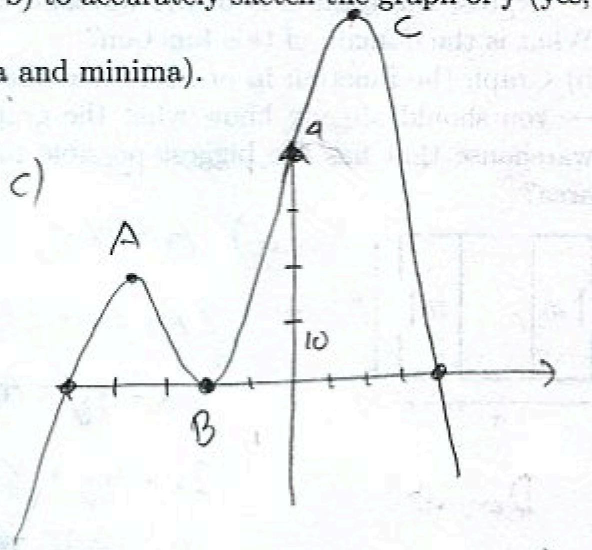
a) Like $-\frac{1}{2}x \cdot x \cdot x^2 = -\frac{1}{2}x^4$ $\cap$

b)

zeros	4	−5	−2
mult.	1	1	2

$f(0) = -\frac{1}{2}(-4)\cdot 5 \cdot 2^2 = 40$

c)

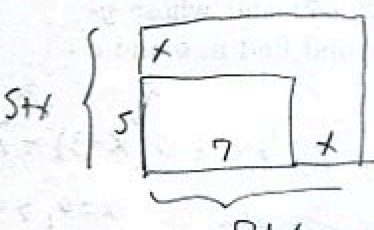


d) Turning points: $(-4.679,\ 16.080377) = A$

$(-2, 0) = B$

$(2.329002,\ 114.75361) = C$

11. (12pts) Starting with a 5 ft×7 ft rectangle, we increased the width and length by the same amount to get a rectangle with area 50 ft². How much was added to the width and length of the 5 × 7 rectangle?



$(5+x)(7+x) = 50$

$x^2 + 12x + 35 = 50$

$x^2 + 12x - 15 = 0$

144 + 60 = 204 = 4·51

$x = \frac{-12 \pm \sqrt{12^2 - 4\cdot 1\cdot(-15)}}{2\cdot 1} = \frac{-12 \pm \sqrt{204}}{2} = \frac{-12 \pm 2\sqrt{51}}{2}$

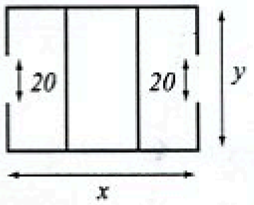
$= \frac{2(-6 \pm \sqrt{51})}{2} = -6 \pm \sqrt{51} = 1.141428$

$-6 - \sqrt{51} < 0$ so cannot be a solution, since $x > 0$

12. (14pts) A logistics company is building a warehouse whose floorplan is below. It has two entrances of width 20 feet. It has budgeted enough money to build 800 feet of walls, and its goal is to maximize the total area of the warehouse.

a) Express the total area of the warehouse as a function of the length of one of the sides. What is the domain of this function?

b) Graph the function in order to find the maximum (no need for the graphing calculator — you should already know what the graph looks like). What are the dimensions of the warehouse that has the biggest possible total area, and what is the biggest possible total area?



a) $A = x \cdot y = (420-2y)y = -2y^2 + 420y$

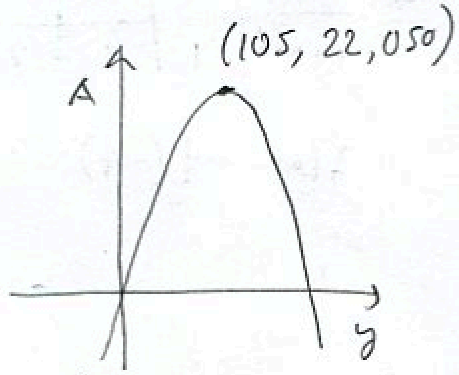
$2x + 2y + 2(y-20) = 800$

$2x + 4y - 40 = 800$

$2x + 4y = 840$

$2x = 840 - 4y$

$x = 420 - 2y$



Domain:

Must have:

$x \geq 0 \quad y \geq 20$

$420 - 2y \geq 0$

$420 \geq 2y$

$210 \geq y$

Domain: $(0, 210]$

$-\frac{b}{2a} = -\frac{420}{2\cdot(-2)} = 105$

$-2 \cdot 105^2 + 420 \cdot 105 = 22,050$

$420 - 2 \cdot 105$

Dimensions: 210×105

Max area: $22,050$

Bonus. (10pts) Find the equation of a parabola whose vertex is $(3, -7)$ and whose y-intercept is 5. One way to approach this is to write $y = ax^2 + bx + c$ and find a, b and c based on the information given.

Plug in $x = 0, y = 5$ in $y = ax^2 + bx + c$

$5 = 0 + 0 + c$

so $c = 5$

$-\frac{b}{2a} = 3$ and

$b = -6a$

$-7 = a \cdot 3^2 + b \cdot 3 + 5$

$-b = -6a$

$-7 = 9a + (-6a) \cdot 3 + 5$

$-7 = 9a - 18a + 5$

$-12 = -9a$

$a = \frac{-12}{-9} = \frac{4}{3}$

$b = -6 \cdot \frac{4}{3} = -8$

$y = \frac{4}{3}x^2 - 8x + 5$

OR:

$y = a(x-h)^2 + k = a(x-3)^2 - 7$

To find a, plug in $x = 0, y = 5$

$5 = a(0-3)^2 - 7$

$5 = 9a - 7$

$12 = 9a$

$a = \frac{12}{9} = \frac{4}{3}$

$y = \frac{4}{3}(x-3)^2 - 7$

$= \frac{4}{3}(x^2 - 6x + 9) - 7$

$= \frac{4}{3}x^2 - 8x + 5$