College Algebra — Exam 2
MAT 140C, Fall 2021 — D. Ivanšić

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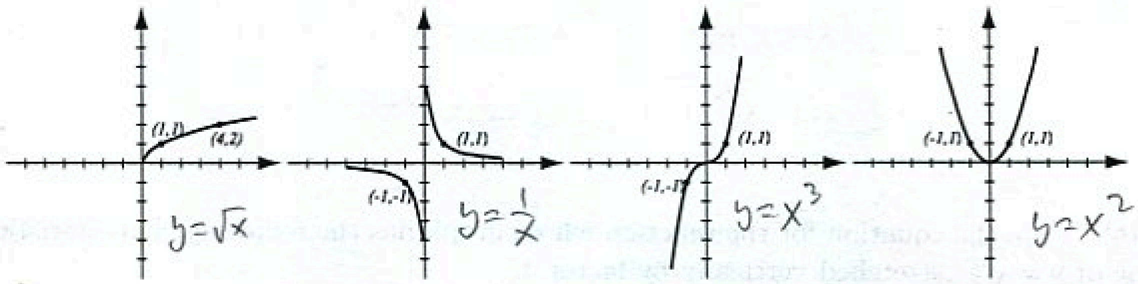
Show all your work!

1. (8pts) The following are graphs of basic functions. Write the equation of the graph under each one.



$y=\sqrt{x}$ $\quad y=\frac{1}{x}$ $\quad y=x^3$ $\quad y=x^2$

2. (20pts) Let $f(x)=\sqrt{8-2x}$, $g(x)=\dfrac{x+1}{x^2+4x-12}$.

Find the following (simplify where possible):

$(f+g)(-4)= f(-4)+g(-4)$

$=\sqrt{8-2\cdot(-4)}+\dfrac{-4+1}{(-4)^2+4(-4)-12}=\sqrt{16}-\dfrac{3}{-12}=4+\dfrac{1}{4}=\dfrac{17}{4}$

$\dfrac{g}{f}(2)=\dfrac{g(2)}{f(2)}=\dfrac{\frac{2+1}{4+8-12}}{\sqrt{8-4}}=\dfrac{\frac{3}{0}}{\sqrt{4}}\Big\}$ not defined

$\dfrac{f}{g}(x)=\dfrac{f(x)}{g(x)}=\dfrac{\sqrt{8-2x}}{\frac{x+1}{x^2+4x-12}}$

$=\dfrac{(x^2+4x-12)\sqrt{8-2x}}{x+1}$

$(f\circ g)(0)= f(g(0))=f\left(\dfrac{0+1}{0+0-12}\right)=f\left(-\dfrac{1}{12}\right)$

$=\sqrt{8-2\cdot\left(-\frac{1}{12}\right)}=\sqrt{8+\frac{1}{6}}=\sqrt{\frac{49}{6}}$

$=\dfrac{7}{\sqrt{6}}$

$(g\circ f)(x)= g(f(x))=g(\sqrt{8-2x})$

$=\dfrac{\sqrt{8-2x}+1}{\sqrt{8-2x}^{\,2}+4\sqrt{8-2x}-12}=\dfrac{\sqrt{8-2x}+1}{8-2x+4\sqrt{8-2x}-12}=\dfrac{\sqrt{8-2x}+1}{-2x-4+4\sqrt{8-2x}}$

The domain of $f-g$ in interval notation

domain f: must have $8-2x\geq 0$

$8\geq 2x$

$4\geq x$

domain g: can't have

$x^2+4x-12=0$

$(x+6)(x-2)=0$

$x=-6,2$

f

overlap

$-6 \quad 2 \quad 4$

$(-\infty,-6)\cup(-6,2)\cup(2,4)$

3. (6pts) Consider the function $h(x) = (x^2+6)^4$ and find **two** different solutions to the following problem: find functions f and g so that $h(x) = f(g(x))$, where neither f nor g are the identity function.

$g(x) = x^2+6$ $\qquad$ $g(x) = x^2$

$f(x) = x^4$ $\qquad$ $f(x) = (x+6)^4$

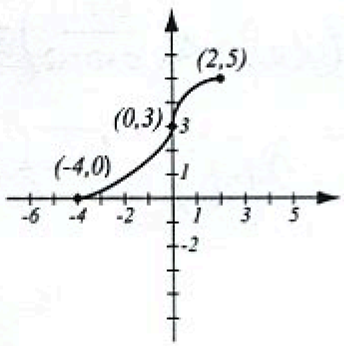
4. (6pts) Write the equation for the function whose graph has the following characteristics:
a) shape of $y = \sqrt{x}$, stretched vertically by factor 4.
b) shape of $y = x^3$, shifted left 5 units, then reflected over the y axis.

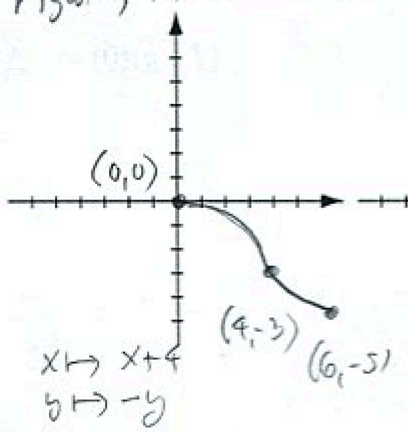
a) $\sqrt{x} \rightsquigarrow 4\sqrt{x}$

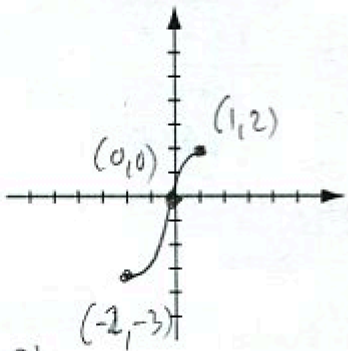
b) $x^3 \rightsquigarrow (x+5)^3 \rightsquigarrow (-x+5)^3$

5. (10pts) The graph of $f(x)$ is drawn below. Find the graphs of $-f(x-4)$ and $f(2x)-3$ and label all the relevant points.

right 4, reflect in x-axis $\qquad$ stretch horiz., factor $= \frac{1}{2}$, down 3







$x \mapsto \frac{1}{2}x$

$y \mapsto y-3$

$x \mapsto x+4$

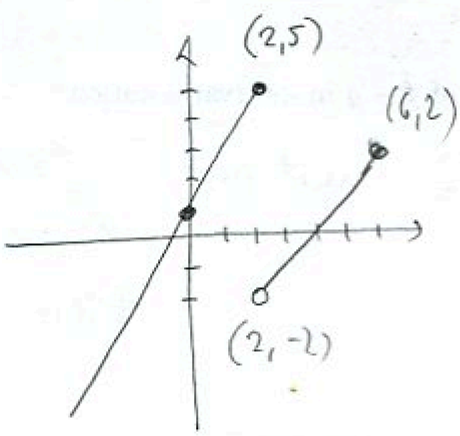
$y \mapsto -y$

6. (8pts) Sketch the graph of the piecewise-defined function:

$$f(x) = \begin{cases} 2x+1, & \text{if } x \leq 2 \\ x-4, & \text{if } 2 < x \leq 6 \end{cases}$$

x	2x+1
2	5
0	1

x	x−4
2	−2
6	2



7. (8pts) Find the values of the piecewise-defined function.

$$f(x) = \begin{cases} 4x-7, & \text{if } -3 < x \le 2 \\ \sqrt{x}, & \text{if } 2 < x \le 10 \\ x^2 - 12x, & \text{if } 10 < x \le 40 \end{cases}$$

$f(20) =$ $20^2 - 12 \cdot 20$
$= 400 - 240 = 160$

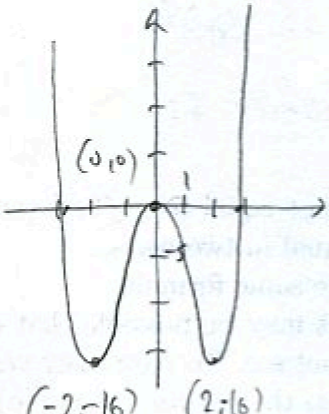
$f(9) =$ $\sqrt{9} = 3$

$f(2) =$ $4 \cdot 2 - 7 = 1$

$f(100) =$ not defined, $100 > 40$

8. (20pts) Let $f(x) = x^4 - 8x^2$ (answer with 6 decimal points accuracy).
a) Use your graphing calculator to accurately draw the graph of f (on paper!). Indicate units on the axes.
b) Determine algebraically whether the function is odd, even, or neither.
c) Verify your conclusion from b) by stating symmetry.
d) Find the local maxima and minima for this function. If there is symmetry, use it to reduce the work here.
e) State the intervals where the function is increasing and where it is decreasing.

a)



b) $f(-x) = (-x)^4 - 8(-x)^2$
$= x^4 - 8x^2 = f(x)$, so even

c) symmetric wrt y-axis

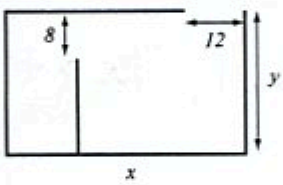
d) Local min is $-16 = f(2)$
$-16 = f(-2)$

Local max is $0 = f(0)$

e) increasing on $(-2, 0)$ and $(2, \infty)$

decreasing on $(-\infty, -2)$ and $(0, 2)$

9. (14pts) A grocery company wishes to build a store that is to have area 6,500 square feet, an entrance door and a door to the rear storage area. To minimize cost, the total length of walls has to be as small as possible.
a) Express the total length of walls of the store as a function of the length of one of the sides x. What is the domain of this function?
b) Graph the function in order to find the minimum. What are the dimensions of the store that has the smallest total wall length?



$A = xy = 6500 \Rightarrow y = \frac{6500}{x}$

$l = x + x - 12 + 2y + y - 8 = 2x + 3y - 20$

$= 2x + 3 \cdot \frac{6500}{x} - 20 = 2x + \frac{19500}{x} - 20$

Domain:
Must have,

$x \geq 12,\ y \geq 8$

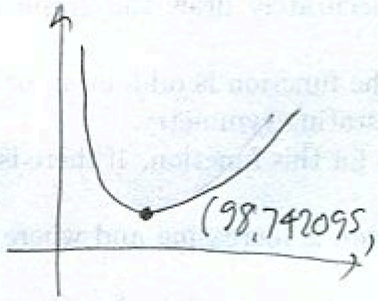
$\frac{6500}{x} \geq 8$

Domain: $[12, 812.5]$

$6500 \geq 8x$

$\frac{6500}{8} \geq x$

$x \leq 812.5$



$\frac{6500}{98.742095}$

Dimensions: $98.742095 \times 65.828054$

Minimal wall length: 374.96835 ft

Bonus. (10pts) Give two functions f and g so that $f(g(x))$ is not equal to $g(f(x))$ (write the formulas for f and g). Verify $f(g(x))$ and $g(f(x))$ are not equal in two ways:
a) Compute $f(g(x))$ and $g(f(x))$ and observe you did not get the same formula.
b) While formulas for $f(g(x))$ and $g(f(x))$ may look different, it may be possible that one can transform one to the other in some difficult way that we cannot see. To rigorously verify that $f(g(x))$ is not equal to $g(f(x))$ give a particular number a so that $f(g(a)) \neq g(f(a))$.

a) $f(x) = x + 3$

$g(x) = x^2$

$\left.\begin{array}{l} f(g(x)) = f(x^2) = x^2 + 3 \\ g(f(x)) = g(x+3) = (x+3)^2 = x^2 + 6x + 9 \end{array}\right\}$ look different

b) $\left.\begin{array}{l} f(g(0)) = 0^2 + 3 = 3 \\ g(f(0)) = 0^2 + 6 \cdot 0 + 9 = 9 \end{array}\right\}$ different.