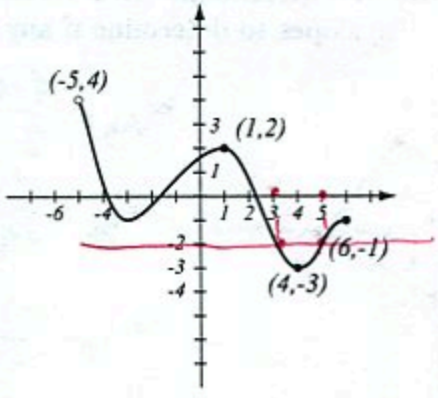
College Algebra — Exam 1
MAT 140C, Fall 2021 — D. Ivanšić

Name: Saul Ocean

Show all your work!

1. (8pts) Use the graph of the function f at right to answer the following questions.



a) Find: $f(4) = -3$ $f(1) = 2$

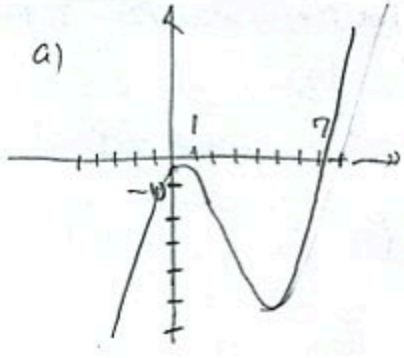
b) What is the domain of f? $(-5, 6]$

c) What is the range of f? $[-3, 4)$

d) What are the solutions of the equation $f(x) = -2$? $x = 3, 5$

2. (10pts) Use your calculator to accurately sketch the graph of $y = x^3 - 8x^2 + 5x - 3$.

a) Draw the graph on paper and indicate units on the axes.

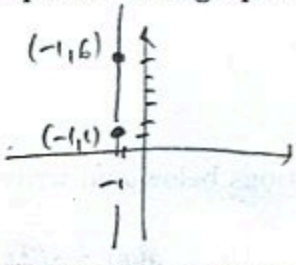


b) Find all the $x-$ and $y-$intercepts (accuracy: 6 decimal points).

b) y-int: $f(0) = -3$

x-int: 7,377373

3. (5pts) Draw the line that passes through points $(-1, 1)$ and $(-1, 6)$. Then write the equation of the line.



It's a vertical line

$x = -1$ is equation

4. (10pts) Find the equation of the line (in form $y = mx + b$) that is parallel to the line $2x - 4y = 5$ and passes through the point $(4, 1)$. Draw both lines.

$2x - 4y = 5$

$2x - 5 = 4y \quad | \div 4$

$y = \frac{2x}{4} - \frac{5}{4}$

$y = \frac{1}{2}x - \frac{5}{4}$

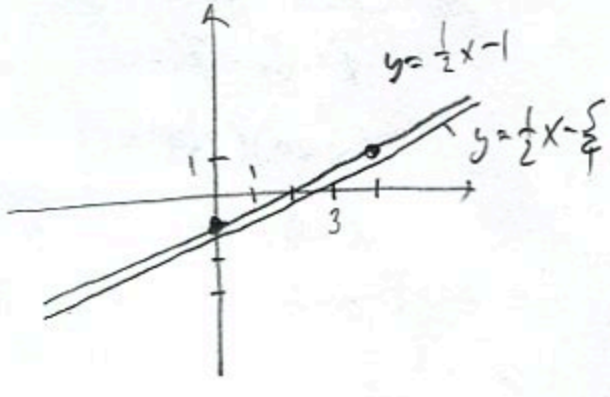
slope $= \frac{1}{2}$

slope of parallel line is $\frac{1}{2}$

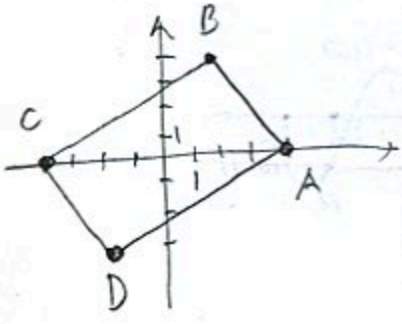
$y - 1 = \frac{1}{2}(x - 4)$

$y = \frac{1}{2}x - \frac{4}{2} + 1$

$y = \frac{1}{2}x - 1$



5. (8pts) Draw the quadrangle with vertices $A = (4,0)$, $B = (2,4)$, $C = (-4,0)$ and $D = (-2,-3)$. Use slopes to determine if any two of its sides are perpendicular.



$m_{AB} = \frac{4-0}{2-4} = \frac{4}{-2} = -2$

$m_{AD} = \frac{-3-0}{-2-4} = \frac{-3}{-6} = \frac{1}{2}$

$m_{BC} = \frac{0-4}{-4-2} = \frac{-4}{-6} = \frac{2}{3}$

$m_{CD} = \frac{-3-0}{-2-(-4)} = \frac{-3}{2} = -\frac{3}{2}$

AB and AD } are
BC and CD } perpendicular
since their slopes are
opposite reciprocal

6. (9pts) Let $f(x) = x^2 - \sqrt{2x-7}$. Find the following (simplify where appropriate).

$f(1) = 1^2 - \sqrt{2\cdot 1 - 7} = 1 - \sqrt{-5}$
not defined

$f(8) = 8^2 - \sqrt{2\cdot 8 - 7} = 64 - \sqrt{9}$
$= 64 - 3 = 61$

$f(4u) = (4u)^2 - \sqrt{2\cdot(4u)-7}$
$= 16u^2 - \sqrt{8u-7}$

$f(x+3) = (x+3)^2 - \sqrt{2(x+3)-7}$
$= x^2 + 6x + 9 - \sqrt{2x+6-7}$
$= x^2 + 6x + 9 - \sqrt{2x-1}$

7. (9pts) Find the domains of the functions below and write them using interval notation.

$f(x) = \dfrac{1}{x^2 - 5x - 36}$

Can't have:
$x^2 - 5x - 36 = 0$
$(x-9)(x+4) = 0$
$x = 9, -4$
$-4 \quad 9$
Domain $= (-\infty, -4) \cup (-4, 9) \cup (9, \infty)$

$g(x) = \sqrt{2x+7}$

Must have
$2x + 7 \geq 0$
$2x \geq -7$
$x \geq -\frac{7}{2}$

$-\frac{7}{2}$

$\left[-\frac{7}{2}, \infty\right)$

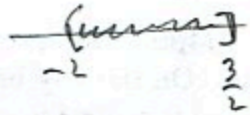
8. (5pts) Solve and write the solution in interval notation.

$4 \le 7 - 2x < 11$ $|-7$

$-3 \le -2x < 4$ $|\div(-2)$

$\frac{-3}{-2} \ge x > \frac{4}{-2}$

$\frac{3}{2} \ge x > -2$



$(-2, \frac{3}{2}]$

9. (10pts) The diameter of a circle has endpoints $(-2, -3)$ and $(4, 1)$.
a) Find the equation of the circle.
b) Draw the circle in the coordinate plane.

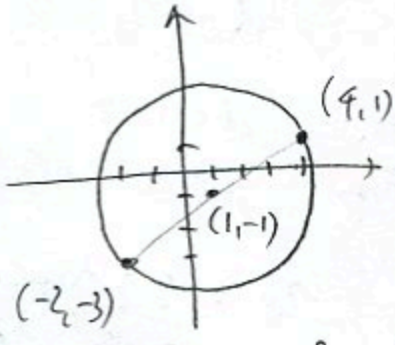
a) center = midpoint of diameter

$= \left(\frac{-2+4}{2}, \frac{-3+1}{2}\right) = \left(\frac{2}{2}, \frac{-2}{2}\right) = (1, -1)$

radius = distance from $(1,-1)$ to $(4,1)$

$= \sqrt{(4-1)^2 + (1-(-1))^2}$

$= \sqrt{3^2 + 2^2} = \sqrt{13}$



$(x-1)^2 + (y-(-1))^2 = \sqrt{13}^2$

$(x-1)^2 + (y+1)^2 = 13$

10. (12pts) An electric company offers two plans to pay for electricity usage:
A) \$60 flat fee that includes 200 kWh, then 12 cents per kWh for usage beyond 200 kWh.
B) \$10 flat fee plus 16 cents per kWh.
Assuming a customer always uses at least 200 kWh of electricity, for which amounts of electricity is plan A better?

x = kwh used

A) cost = $60 + 0.12(x - 200)$

B) cost = $10 + 0.16x$

Asking when $A \le B$

$60 + 0.12(x-200) \le 10 + 0.16x$

$60 + 0.12x - 24 \le 10 + 0.16x$ $|$

$36 + 0.12x \le 10 + 0.16x$ $|-10$

$26 + 0.12x \le 0.16x$ $|-0.12x$

$26 \le 0.04x$ $|\div 0.04$

$650 \le x$

when customer uses more than 650 kWh plan A is better

11. (14pts) Because she was afraid to be late, Fiona rushed to a concert and got there in 2 hours. On the way back, she drove 9mph slower, so it took her a quarter of an hour longer.
a) How fast did Fiona drive to and from the concert?
b) How far did she drive to the concert?

dist. rate time

$d, r, 2$ →

← $d, r-9, 2.25$

To: $d = r \cdot 2$

From: $d = (r-9) \cdot 2.25$

$2r = 2.25(r-9)$

$2r = 2.25r - 20.25$ $\quad |+20.25$

$2r + 20.25 = 2.25r$ $\quad |-2r$

$20.25 = 0.25r$ $\quad |\div 0.25$

$81 = r$

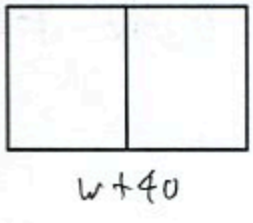
a) drove to concert: 81 mph

From concert: 72 mph

b) Distance to concert is

$d = 81 \cdot 2 = 162$ miles

Bonus (10pts) The length of a rectangular field is 40 feet more than the width. A farmer used 470 feet of fencing to enclose the field and divide it into two parts, as in the picture. What are the dimensions of the field?



Amount of fence used: $3w + 2(w+40) = 470$

$3w + 2w + 80 = 470$ $\quad |-80$

$5w = 390$

$w = \frac{390}{5} = 78$

Field is 78×118