1. (30pts) Greek gods are considering what scourge to unleash on humanity (because, as it is, we have it so good!). The proposals are Fires, Hurricanes, Pandemic, and Flooding, which the gods rank. They will decide on the scourge based on these rankings.

Votes	5	2	2	3	8	3	3	1	= votes: 27
1st	Fi	Fi	Hu	Hu	Pa	Fl	Fl	Fl
2nd	Pa	Fl	Fl	Pa	Fl	Fi	Hu	Hu
3rd	Hu	Hu	Fi	Fi	Fi	Pa	Fi	Pa
4th	Fl	Pa	Pa	Fl	Hu	Hu	Pa	Fi

a) Which choice wins the vote in a plurality election?
b) Which choice wins the vote in a plurality election with elimination?
c) Which choice is the pairwise comparison winner?
d) Which choice is the winner using Borda's method? Perform the check on the sum of Borda points.

a) Fi $5+2=7$ —— b) $7 = 7$ elim

Fl $3+3+1=7$ ——→ $7+2 = 9$ ——→ $9+2 = 11$

Hu $2+3=5$ elim.

Pa 8 wins ——→ $8+3 = 11$ ——→ $11+5 = 16$ win

c) Fi vs Fl   Fi vs Hu   Fi vs Pa
$5+2+3$ $3+3+1+2+8$ $5+2+8+3$ $2+3+3+1$ $5+2+2+3+3$ $8+3+1$
$= 10$ $(= 17)$ $(= 18)$ $= 9$ (15) $≅12$

Fl vs Hu   Fl vs Pa   Hu vs Pa
$3+3+1+2+8$ $2+3+5$ $3+3+1+2+2$ $8+5+3$ $2+3+3+1+12$ $8+3+5$
$(= 17)$ $= 10$ $= 11$ (16) $≅ 11$ $(≅ 16)$

Fi	Fl	Hu	Pa
2	2		2

It's a tie

d) Fi: $7·4 + 3·3 + 16·2 + 1·1 = 70$

Fl: $7·4 + 12·3 + 0·2 + 8·1 = 72$ wins

Hu: $5·4 + 4·3 + 7·2 + 11·1 = 57$

Pa: $8·4 + 8·3 + 4·2 + 7·1 = 71$

$\overline{\qquad\qquad 270}$

27 voters × 10 pts each = 270

$(1+2+3+4)$

**2.** (16pts) For each of the following graphs:
a) State and justify whether it has an Euler path.
b) State and justify whether it has an Euler circuit.
c) If it has either an Euler path or a circuit, indicate it on the graph. Use arrows and number the edges to indicate how the Euler path or circuit goes around the graph.
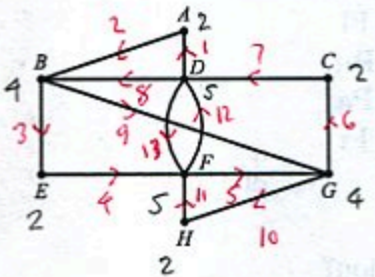
Exactly two odd vertices

Has Euler path

No Euler circuit
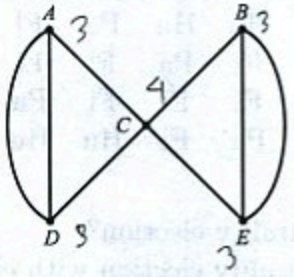
More than 2 odd vertices:

no Euler path

no Euler circuit
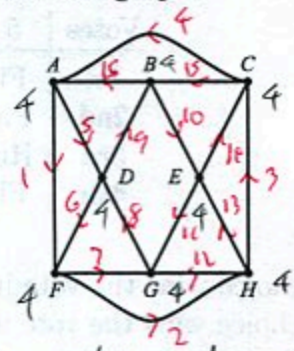
All vertices even:

Has Euler circuit

(so it has an Euler path, too, that starts and ends at same vertex.

3. (14pts) A mail carrier has to deliver mail to the neighborhood shown in the picture. The mail carrier always walks one row of houses on one side of the street at a time.
a) Draw a graph that models the neighborhood.
b) Can the mail carrier deliver the mail to every house in the neighborhood without walking by any row of houses twice? If so, display the route.
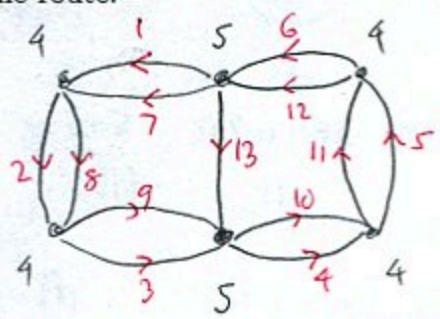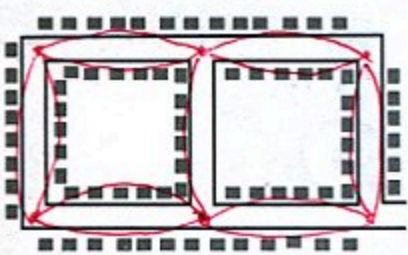
b) Yes. Associated graph has two odd vertices, so has Euler circuit.