

Final answers should have accuracy to 6 decimal places (or 4 decimal places for table-derived answers). Show some work how the mean and standard deviation are computed. Giving only the answer will bring you few points.

1. (15pts) A small car repair shop tracks how busy it is by counting how many cars it services each day. The results are below.

- a) Find the range of the data.
 b) Find the mean of the data.
 c) Find the standard deviation of the data.

Cars repaired	Frequency (days)
3	4
4	3
5	10
6	9
7	5
	<u>31</u>

a) Range = $7 - 3 = 4$

b) $\bar{x} = \frac{4 \cdot 3 + 3 \cdot 4 + 10 \cdot 5 + 9 \cdot 6 + 5 \cdot 7}{31} = \frac{163}{31}$

$\bar{x} = 5.258065$

c) $4 \cdot (3 - 5.258065)^2 + 3 \cdot (4 - 5.258065)^2 + 10 \cdot (5 - 5.258065)^2 + 9 \cdot (6 - 5.258065)^2 + 5 \cdot (7 - 5.258065)^2 = 45.93...$

$s = \sqrt{\frac{45.93...}{30}} = 1.237410$

2. (15pts) The longevity of heat pump compressors (time in service until they have to be replaced) has been found to have mean 11 years, with standard deviation 2.5 years. Use the 68-95-99.7 rule (draw a picture) to find the percentage of compressors that lasted:

- a) between 8.5 and 11 years

$\frac{68}{2} = 34\%$

- b) under 6 years

$\frac{95}{2} = 47.5, 50 - 47.5 = 2.5\%$

- c) over 18.5 years

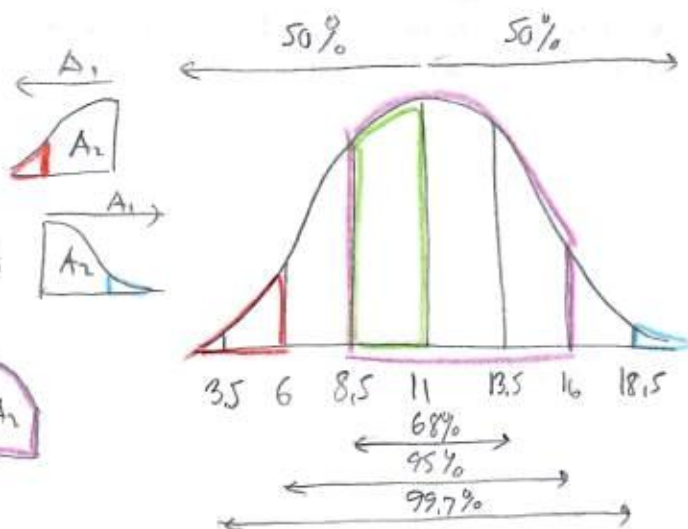
$\frac{99.7}{2} = 49.85, 50 - 49.85 = 0.15\%$

- d) between 8.5 and 16 years.

$\frac{68}{2} = 34, \frac{95}{2} = 47.5$

$34 + 47.5 = 81.5\%$

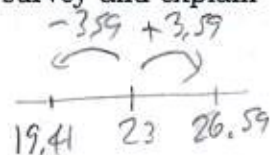
$A_1 + A_2$



3. (5pts) A survey of 774 adults found that 23% of them have no intention of getting vaccinated against the COVID-19 virus. Find the margin of error of this survey and explain what it means.

$$\frac{1}{\sqrt{774}} = 0.0359443$$

Margin of error = 3.59%



There is a 95% probability that the true percentage of people with no intention of being vaccinated is between 19.41% and 26.59%

4. (5pts) Two students took standardized exams. Student A scored 35 points on an exam with mean 32 and standard deviation 4, and student B scored 63 points on an exam with mean 58 and standard deviation 6. Use z-scores to determine which student did better.

$$z_A = \frac{35-32}{4} = \frac{3}{4} = 0.75$$

Scored 0.75 stand. dev. above mean

$$z_B = \frac{63-58}{6} = \frac{5}{6} \approx 0.83$$

scored 0.83 stand. dev. above mean

Student B better

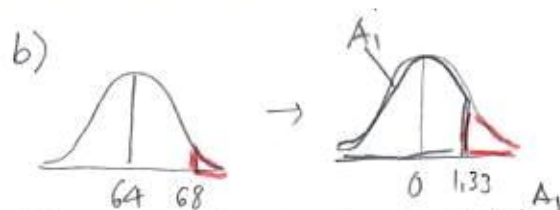
5. (20pts) The speed of vehicles on an expressway during rush hour is normally distributed with mean 64 mph and standard deviation 3 mph. Draw a picture showing which area you are computing as you answer:

- What percentage of vehicles have speed under 60 mph?
- What percentage of vehicles have speed above 68 mph?
- What is the percentile of a vehicle traveling at 55 mph? What does this mean?
- What is the probability that a random vehicle is driving between 56 and 62 mph?



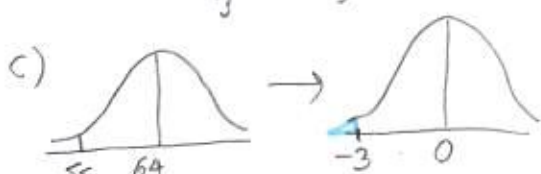
$$P(X \leq 60) = P(Z \leq -1.33) = 0.0918, \boxed{9.18\%}$$

$$\frac{60-64}{3} = -\frac{4}{3} = -1.33$$



$$P(X \geq 68) = P(Z \geq 1.33) = 1 - 0.9082 = 0.0918$$

$$\frac{68-64}{3} = 1.33$$

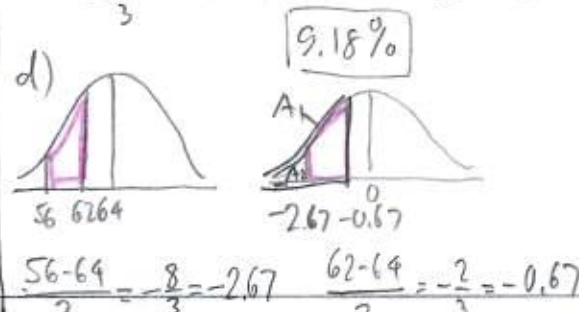


$$P(X \leq 55) = P(Z \leq -3) = 0.0044$$

$$\frac{55-64}{3} = -3$$

0.44 percentile

0.13% of vehicles are slower than a car traveling at 55 mph



$$P(56 \leq X \leq 62) = P(-2.67 \leq Z \leq -0.67) = 0.2514 - 0.0038 = \boxed{0.2476}$$

$$\frac{56-64}{3} = -\frac{8}{3} = -2.67$$

$$\frac{62-64}{3} = -\frac{2}{3} = -0.67$$