Mathematical Concepts — Joysheet 7 MAT 117, Fall 2021 — D. Ivanšić

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Covers: 11.7, 11.8 *Show all your work!*

1. (14pts) Avid movie-watcher Martin watches two movies per week. For a movie Martin watches, the likelihood he finds it satisfying is 20%, that he finds it mediocre is 45% and that he finds it disappointing is 35%. Assume Martin's liking of the two movies are independent of each other. What is the probability that
a) The first movie is mediocre and the second is satisfying?
b) Martin finds one movie disappointing and one mediocre?
c) At least one of the movies is mediocre or better?

a) $P(\text{1st med. AND 2nd sat.}) = P(\text{1st med}) \cdot P(\text{2nd sat.}) = 0.45 \cdot 0.2 = 0.09$

b) $P(\text{one is dis. AND one is med.}) = P((\text{1st is dis AND 2nd is med.}) \text{ OR } (\text{1st is med AND 2nd is dis.}))$
$= P(\text{1st is dis. AND 2nd is med.}) + P(\text{1st is med. AND 2nd is dis.})$
$= P(\text{1st is dis.}) \cdot P(\text{2nd is med.}) + P(\text{1st is med.}) \cdot P(\text{2nd is dis.})$
$= 0.35 \cdot 0.45 + 0.45 \cdot 0.35 = 0.1575 \cdot 2 = 0.315$

c) $P(\text{At least one is med. or sat.}) = 1 - P(\text{both are dis.})$
$= 1 - P(\text{1st is dis. AND 2nd is dis.}) = 1 - P(\text{1st is dis}) \cdot P(\text{2nd is dis}) = 1 - 0.35 \cdot 0.35$
$= 1 - 0.1225 = 0.8775$

2. (14pts) Three kids are picked from a group of four 3rd-graders, seven 4th-graders and three 5th-graders. What is the probability that: $(4+7+3 = 14 \text{ kids})$
a) The second kid is a 4th-grader, given that the first one was a 4th-grader?
b) The first kid is a 3rd-grader, the second a 4-th grader and the third a 5th-grader?
c) All three are not 3rd-graders?
d) At least one is a 5-th grader?

a) $P(\text{2nd is 4th g.} \mid \text{1st is 4th g.}) = \frac{6}{13}$

b) $P(\text{1st is 3rd g. AND 2nd is 4th g. AND 3rd is 5th g.})$
$= P(\text{1st is 3rd g.}) \cdot P(\text{2nd is 4th g.} \mid \text{1st is 3rd g.}) \cdot P(\text{3rd is 5th g.} \mid \text{1st is 3rd g. AND 2nd is 4th g.})$
$= \frac{4}{14} \cdot \frac{7}{13} \cdot \frac{3}{12} = \frac{1}{26}$

c) $P(\text{all three are not 3rd g}) = P(\text{1st not 3rd g. AND 2nd not 3rd g. AND 3rd not 3rd g.})$
$= P(\text{1st not 3rd g.}) \cdot P(\text{2nd not 3rd g.} \mid \text{1st not 3rd g.}) \cdot P(\text{3rd not 3rd g.} \mid \text{1st AND 2nd not 3rd g.})$
$= \frac{10}{14} \cdot \frac{9}{13} \cdot \frac{8}{12} = \frac{30}{91}$

like c)

d) $P(\text{At least one is 5-th g.}) = 1 - P(\text{all three not 5-th g}) = 1 - \frac{11}{14} \cdot \frac{10}{13} \cdot \frac{9}{12} = 1 - \frac{165}{364} = \frac{199}{364}$

3. (10pts) The table shows the varieties of potato chips on a convenience store shelf with respect to flavoring and the style of frying. What is the probability that a random bag of chips:
a) is kettle-cooked jalapeno?
b) is standard-fried?
c) is plain?
d) is plain, given it is standard-fried?
e) is not vinegar, given it is kettle-cooked?

Type	Standard	Kettle	Total
Plain	10	14	24
Jalapeno	6	12	18
Vinegar	9	8	17
Total	25	34	59

a) $\frac{12}{59}$ b) $\frac{25}{59}$ c) $\frac{24}{59}$ d) $\frac{10}{25} = \frac{2}{5}$ e) $\frac{26}{34} = \frac{13}{17}$

4. (10pts) A game of chance is played in this way: a player pays \$5 and draws a card. If the card is an ace, the player gets \$55.
a) Determine the player's expected value.
b) If the player plays this game 39 times, how much do they expect to win or lose?
c) What is the fair price of this game?

a)

Outcomes

net gain	prob
\$50	$\frac{4}{52} = \frac{1}{13}$
-5	$\frac{48}{52} = \frac{12}{13}$

$E = 50 \cdot \frac{1}{13} + (-5) \cdot \frac{12}{13} = \frac{50-60}{13} = -\frac{10}{13} \approx -0.77$

b) $\cancel{39}^{3} \cdot (-\frac{10}{\cancel{13}}) = -30$ Player expects to lose \$30

c) Fair price $= E +$ cost to play $= -\frac{10}{13} + 5 = \frac{-10+65}{13} = \frac{55}{13} \approx 4.23$

5. (12pts) A company issues insurance for people who rent their dwelling, covering damage to the renters' possessions in case of fire or theft. Based on past claims, the company's estimates of chances of annual payouts are in the table.
a) Calculate expected value of a renter's claim.
b) How much should the company charge for a 1-year policy to break even on claim costs?
c) How much should the company charge to make a profit of \$70 per policy?

Amount of claim	Probability
0	0.71
500	0.14
2,000	0.08
5,000	0.05
10,000	~~0.02~~ 0.02

(all have to add up to 1)

a) $0 \cdot 0.71 + 500 \cdot 0.14 + 2000 \cdot 0.08 + 5000 \cdot 0.05 + 10{,}000 \cdot 0.02$

$= 70 + 160 + 250 + 200 = \$680$

b) Company should charge \$ 680

c) It should charge $680 + 70 = 750$