

**Mathematical Concepts — Joysheet 6**  
**MAT 117, Fall 2021 — D. Ivanšić**

**Name:** Saul Ocean  
*Covers: 11.4, 11.6 Show all your work!*

1. (10pts) Current member states of the European Union have been classified into groups in the table according to their current populations (in millions) and time of joining the union.

Joined EU	Pop. 0-6	Pop. 6-20	Pop. > 20
Before 2000	4	6	4
After 2000	8	4	1

Total countries: 27

a)  $\frac{8+4+1}{27} = \frac{13}{27}$       b)  $\frac{4+1}{27} = \frac{5}{27}$

c)  $\frac{4}{27}$       d)  $\frac{8+1}{27} = \frac{9}{27} = \frac{1}{3}$

If a random member country of the EU is selected, what is the probability that it

- a) joined after 2000?
- b) has population greater than 20 million?
- c) joined before 2000 and has population below 6 million?
- d) joined after 2000 and does not have population between 6 and 20 million?

2. (20pts) Write the probabilities and odds against and in favor of the following events (show any work needed below):

Event	probability	odds against	odds in favor
a) Drawing an ace from a deck of cards	$\frac{4}{52} = \frac{1}{13}$	12 to 1	1 to 12
b) Pulling a red sock without looking from a bag containing 4 red and 7 white socks	$\frac{4}{11}$	7 to 4	4 to 7
c) Getting exactly one tail on three coin tosses	$\frac{3}{8}$	5 to 3	3 to 5
d) Drawing a face card or a black 9 from a deck of cards	$\frac{7}{26}$	19 to 7	7 to 19
e) Getting a sum of 6 or getting a 4 on at least one of the dice on a roll of two dice	$\frac{7}{18}$	11 to 7	7 to 11

c) 8 outcomes, exactly one tail: THH, HTH, HHT

d)  $P(\text{face or black 9}) = P(\text{face}) + P(\text{black 9}) = \frac{4 \cdot 3}{52} + \frac{2}{52} = \frac{14}{52} = \frac{7}{26}$

mutually exclusive

e)  $P(\text{sum}=6 \text{ or } 4 \text{ on } \geq 1 \text{ die}) = P(\text{sum}=6) + P(4 \text{ on } \geq 1 \text{ die}) - P(\text{sum}=6 \text{ AND } 4 \text{ on } \geq 1 \text{ die})$   
 $= \frac{5}{36} + \frac{11}{36} - \frac{2}{36}$

36 outcomes, in overlap

sum=6 (1,5), (2,4), (3,3), (4,2), (5,1)

at least 1 is a 4:  $\frac{4}{36}$  or  $\frac{4}{36}$  or  $\frac{4}{36}$   
 (4,1), (1,4), (4,4)  
 $5+5+1=11$

$= \frac{14}{36} = \frac{7}{18}$

3. (4pts) The odds against Wilma buying a cantaloupe on her weekly visit to the grocery are 7-to-12.

a) What is the probability Wilma buys a cantaloupe on her weekly visit to the grocery?

$$\frac{12}{19}$$

b) What is the probability Wilma doesn't buy a cantaloupe on her weekly visit to the grocery?

$$\frac{7}{19}$$

4. (4pts) In a typical year in Seattle, Washington, 42% of days are sunny.

a) What are the odds in favor of choosing a sunny day in Seattle?

$$a) \frac{0.42}{0.58} = \frac{42}{58} = \frac{21}{29} \quad 21 \text{ to } 29$$

b) What are the odds against choosing a sunny day in Seattle?

$$b) \quad 29 \text{ to } 21$$

5. (8pts) Among 65 cars on a dealer's lot, 22 are white, 17 have heated front seats, and 8 are white and have heated front seats. What is the probability that a randomly chosen car from the lot

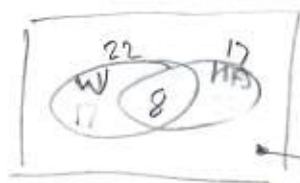
a) is white or has heated front seats?

b) is neither white nor does it have heated front seats?

$$a) P(W \text{ or HFS}) = P(W) + P(HFS) - P(W \text{ AND HFS}) = \frac{22}{65} + \frac{17}{65} - \frac{8}{65} = \frac{31}{65}$$

$$b) P(\text{neither white nor HFS}) = P(\text{NOT}(white \text{ or HFS}))$$

$$= 1 - \frac{31}{65} = \frac{65-31}{65} = \frac{34}{65}$$



neither white nor HFS

6. (14pts) Three dice are rolled: red, white and green. Write the number of outcomes of this experiment and then compute the probability that

a) exactly one of the dice came up with a 5.

b) at least one of the dice came up with a 2.

c) the sum on the red and white dice is 5 or the sum on the white and green dice is 6.

$$a) P(\text{exactly one is 5}) = P(\text{only R is 5 OR only W is 5 OR only G is 5})$$

$$= P(\text{only R is 5}) + P(\text{only W is 5}) + P(\text{only G is 5})$$

$$R \text{ is } 5: \frac{5}{216} \times \frac{5}{216} \times \frac{5}{216} = \frac{1 \cdot 5 \cdot 5}{216} + \frac{5 \cdot 1 \cdot 5}{216} + \frac{5 \cdot 5 \cdot 1}{216} = \frac{8 \cdot 25}{216} = \frac{25}{72}$$

$$b) P(\text{at least one is 2}) = 1 - P(\text{none are 2}) = 1 - \frac{5 \cdot 5 \cdot 5}{216} = \frac{216-125}{216} = \frac{91}{216}$$

$$c) P(R+W=5 \text{ OR } W+G=6) = P(R+W=5) + P(W+G=6) - P(R+W=5 \text{ AND } W+G=6)$$

$R+W=5$  (4 ways): (1,4), (2,3), (3,2), (4,1)  $\frac{4 \cdot 6}{216}$   
 $W+G=6$  (5 ways): (1,5), (2,4), (3,3), (4,2), (5,1)  $\frac{6 \cdot 5}{216}$   
 $R+W=5 \text{ AND } W+G=6$  (4 ways): (1,4,2), (2,3,3), (3,2,4), (4,1,5)  $\frac{4}{216}$

$$= \frac{4 \cdot 6}{216} + \frac{6 \cdot 5}{216} - \frac{4}{216} = \frac{50}{216} = \frac{25}{108}$$