Mathematical Concepts — Joysheet 3
MAT 117, Fall 2021 — D. Ivanšić

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Covers: 8.4–8.5 *Show all your work!*

1. (8pts) Sally would like to have \$2000 for a good electric guitar. How much should she deposit now in an account bearing 3.46%, compounded quarterly, in order to have the desired amount in two and a half years? How much of the \$2000 came from interest?

$A = P\left(1+\frac{r}{n}\right)^{nt}$

$2000 = P\left(1+\frac{0.0346}{4}\right)^{4\cdot 2.5}$

$2000 = P \cdot 1.089\_$

$P = \frac{2000}{1.089\_} = 1834.95$

$2000 - 1834.95 = 165.05$ came from interest.

2. (6pts) Bank of Guru is offering a 1.33% interest rate on an account that is compounded weekly, while Swami Bank has an account at 1.36%, compounded quarterly. Which account is the better deal?

$APY = \left(1+\frac{r}{n}\right)^{n} - 1$

Bank of Guru APY: $\left(1+\frac{0.0133}{52}\right)^{52} - 1 = 0.0133871$, 1.33871%

Bank of Swami APY: $\left(1+\frac{0.0136}{4}\right)^{4} - 1 = 0.0136695$, 1.36695% better

3. (10pts) Tiana deposited \$1500 into an account bearing 3.3%, compounded quarterly. After two years, she needed some money and withdrew \$800. How much is in the account five years from her deposit?

$A = P\left(1+\frac{r}{n}\right)^{nt}$

$A = 1500\left(1+\frac{0.033}{4}\right)^{4\cdot 2} = 1500 \cdot 1.067\_ = 1601.91$

After withdrawal, $1601.91 - 800 = 801.91$ is left in account, which sits there for another 3 years.

$A = 801.91\left(1+\frac{0.033}{4}\right)^{4\cdot 3} = 801.91 \cdot 1.103\_ = 885.00$

4. (10pts) To save for a car in four years (approximate cost \$20,000), you make monthly deposits into an account bearing 4.5%, compounded monthly.
a) How much should you deposit every month to reach your goal?
b) How much would you earn in interest over the four years?

$A = P\,\dfrac{\left(1+\frac{r}{n}\right)^{nt}-1}{\frac{r}{n}}$

$P = \dfrac{20000}{52.48} = 381.07$

a) $20{,}000 = P \cdot \dfrac{\left(1+\frac{0.045}{12}\right)^{12\cdot 4}-1}{\frac{0.045}{12}}$

b) Total interest $= 20{,}000 - 381.07 \cdot 12 \cdot 4$
$= 1708.64$

$20{,}000 = P \cdot 52.48$

5. (16pts) At age 22, Juan inherited a retirement account with \$66,000 in it, which he decided to leave alone. At age 29, he found a good job so had money to start adding to the retirement account, depositing \$400 every month. Suppose the account grew all the time at rate 8.5%, compounded monthly.
a) How much is in the account when Juan is 45?
b) How much of it was from deposits and inheritance, and how much from interest?

a) Treat inheritance and Juan's deposits as separate accounts, add amounts.

$A_1 = 66{,}000\left(1+\frac{0.085}{12}\right)^{12\cdot 23}$ (45−22)

$= 66{,}000 \cdot 7.015..$

$= 463{,}016.79$

$A_2 = 400 \cdot \dfrac{\left(1+\frac{0.085}{12}\right)^{12\cdot 16}-1}{\frac{0.085}{12}}$ (45−29)

$= 400 \cdot 406.24..$

$= 162{,}497.48$

$A_1 + A_2 = 625{,}514.27$

amount in account at age 45

b) $66000 + 400 \cdot 12 \cdot 16 = 142{,}800$ total deposits and inheritance

$625{,}514.27 - 142{,}800 = 482{,}714.27$ From interest

6. (10pts) You bought a shares of stock when they cost \$3.56 and sold them in three years at \$8.48. Assuming annual compounding, at what annual rate did this investment grow?

$A = P\left(1+\frac{r}{n}\right)^{nt}$

$8.48 = 3.56\left(1+\frac{r}{1}\right)^{1\cdot 3}$

$8.48 = 3.56\,(1+r)^3 \;\big|\; \div 3.56$

$\dfrac{8.48}{3.56} = (1+r)^3$

$2.38.. = (1+r)^3 \;\big|\; \sqrt[3]{\ }$

$2.38..^{\frac{1}{3}} = 1+r \;\big|\; -1$

$2.38..^{\frac{1}{3}} - 1 = r$

$r = 0.335515$, or 33.5515%