

**Mathematical Concepts — Exam 1**  
**MAT 117, Fall 2021 — D. Ivanišić**

**Name:** \_\_\_\_\_  
*Show all your work!*

$$I = Prt \quad A = P(1 + rt) \quad A = P \left(1 + \frac{r}{n}\right)^{nt} \quad A = P \frac{\left(1 + \frac{r}{n}\right)^{nt} - 1}{\frac{r}{n}} \quad P = PMT \frac{1 - \left(1 + \frac{r}{n}\right)^{-nt}}{\frac{r}{n}} \quad Y = \left(1 + \frac{r}{n}\right)^n - 1$$

1. (6pts) 13 is 32 percent of which number?
2. (8pts) How much should be deposited now in an account bearing 4.18% interest, compounded quarterly, in order to have \$2000 in five years?
3. (10pts) A pair of sneakers with original price \$99 is on sale with an 18% reduction in price. A shopper bought the discounted pair in Kentucky, where sales tax is 6%. What is the total amount she paid for the sneakers?
4. (8pts) You took out a loan of \$500 with simple interest rate 10% and repaid it with \$562.50. How long did you have the loan?

5. (14pts) Single mom Fiona, who has two children, files income taxes using the “head of household” filing status. She earned \$54,000 in wages, \$690 in interest, and deposited \$7,000 into a retirement account; she paid \$6,200 in mortgage interest, \$1,400 in property taxes, \$3,100 in state income taxes and donated \$300 to charity.

a) Find Fiona’s gross income and adjusted gross income.

b) Use the table below to first determine Fiona’s taxable income (don’t forget the exemptions) and then find the tax on this income.

Tax rate	Head of Household
10%	up to \$13,250
15%	\$13,250 to \$50,400
25%	\$50,400 to \$130,150
28%	\$130,150 to \$210,800
33%	\$210,801 to \$413,350
35%	\$413,351 to \$441,000
39.6%	above \$441,000
Standard Deduction	\$9,300
Exemptions (per person)	\$4,050

6. (14pts) Angelo would like to save for a \$14,000 motorcycle.

a) How much should he deposit every week into an account with 4.75% interest, compounded weekly, in order to have \$14,000 in five years?

b) How much of the final amount is from deposits and how much from interest?

7. (32pts) True story: physician assistant Hayley Arceneaux spent three days in orbit on a SpaceX spacecraft. Made-up part: to house all the memorabilia connected to this flight, she decided to build an addition to her house costing \$104,000, financing it with a 15-year loan at interest rate 2.34%, compounded monthly.

a) What is her monthly payment on the loan?

b) What are her total payments over the course of the loan? How much of this amount is for interest?

c) How much of her first payment goes toward interest, and how much towards the principal?

d) What is the balance on the loan after 9 years?

8. (8pts) Bologna Bank is offering a 2.34% interest rate on an account that is compounded daily, while Salami Savings has an account with interest rate 2.32%, compounded quarterly. Which account is the better deal?

**Bonus.** (10pts) Darius invested \$5000 in a mutual fund. Six years later, this investment had value \$13,500. Assuming annual compounding, at what annual rate did this investment grow?

**Mathematical Concepts — Exam 2**  
**MAT 117, Fall 2021 — D. Ivanišić**

**Name:** \_\_\_\_\_  
*Show all your work!*

$$\frac{a}{b} = \frac{P(E)}{1-P(E)} \quad P(E) = \frac{a}{a+b} \text{ where odds in favor of } E \text{ are } a : b \quad P(B | A) = \frac{n(A \text{ and } B)}{n(A)}$$
$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$
$$P(A \text{ or } B) = P(A) + P(B) \text{ (if } A \text{ and } B \text{ are mutually exclusive)}$$
$$P(A \text{ and } B) = P(A) \cdot P(B | A) \quad P(A \text{ and } B) = P(A) \cdot P(B) \text{ if } A \text{ and } B \text{ are independent}$$
$$E = P_1 \cdot A_1 + P_2 \cdot A_2 + \cdots + P_n \cdot A_n$$

**1.** (6pts) An exam has 11 multiple-choice questions. The first five have three answers each, and the remaining six have four answers each. If every question is answered with exactly one choice, in how many ways can the exam be answered?

**2.** (6pts) An ice cream store has 13 flavors of ice cream, 5 types of cones, 6 varieties of toppings and 4 types of syrup. Assuming you are getting one scoop of ice cream in a cone and the toppings and syrup are optional, how many different ice cream treats can you build?

**3.** (10pts) The table shows the inventory of a car dealer's lot with respect to age and type. What is the probability, in fraction form, that a random car from this lot:

- a) is an SUV?
- b) is a new sedan?
- c) is an SUV or a pick-up?
- d) is a sedan, given it is new?
- e) is new, given it is a pick-up?

Type	New	Used	Total
Sedan	5	11	
SUV	13	25	
Pick-up	8	6	
Total			

4. (4pts) Suppose the odds against it raining over a weekend are 5-to-9.

- a) What is the probability of it raining?
- b) What is the probability of it not raining?

5. (20pts) Write the probabilities and odds against and in favor of the following events (you can show any work needed below):

Event	probability	odds against	odds in favor
a) Getting a 1 or a 4 on a roll of a die			
b) Drawing a red queen from a deck of cards			
c) Getting sum 8 on a roll of two dice			
d) Getting the same number on both dice on a roll of two dice			
e) Getting a tail on the first or third toss of three coin tosses			

6. (12pts) In a collection of draw-by-number apps, 76% are free with ad support and 35% can be purchased ad-free. Suppose all apps in this collection are free with ad support or can be purchased ad-free. If an app is randomly selected from this collection what is the probability it

- a) is free with ad support and can be purchased ad-free?
- b) lacks at least one of the features above?

7. (12pts) A game of chance works like this: a player pays \$3 to roll a die. If 1 or 3 comes up, the player wins \$2. If 2, 4 or 5 comes up, the player wins nothing. If 6 comes up, the player wins \$10.

- a) Determine the player's expected value.
- b) If the player plays this game 60 times, how much do they expect to win or lose?
- c) What is the fair price of this game?

8. (14pts) In these supply-chain burdened days, you can't always find your preferred product at the store. Suppose the probability that you obtain store-brand cream cheese on one visit to a grocery store is 60%. Assume that availability of this cheese on different visits are independent events. What is the probability that:

- a) on two trips to the store you find the cheese both times?
- b) on three trips to the store, you find the cheese at least once?
- c) on three trips to the store, you find the cheese the second and third time, but not the first?

9. (16pts) Two cards are drawn from a deck with 52 cards. What is the probability that
- a) The second one is a five, if the first one is a king?
  - b) The first is a seven, and the second a picture card?
  - c) Exactly one card is an ace?

**Bonus.** (10pts) Kids are throwing balls at a wall they made of big foam bricks. Suppose on one throw they hit the wall with probability 0.7. If a single ball hits the wall, the wall collapses with probability 0.2, and if two balls hit the wall, it collapses with probability 0.6. What is the probability that the wall collapses after two throws? *Hint:  $P(\text{collapse}) = P(\text{exactly one ball hits AND collapses}) + P(\text{both balls hit AND collapses})$*



**Mathematical Concepts — Exam 3**  
**MAT 117, Fall 2021 — D. Ivanić**

**Name:** \_\_\_\_\_  
*Show all your work!*

Final answers should have accuracy to 6 decimal places (or 4 decimal places for table-derived answers). Show some work how the mean and standard deviation are computed. *Giving only the answer will bring you few points.*

$\text{midrange} = \frac{\text{lowest value} + \text{highest value}}{2} \quad \text{range} = \text{highest value} - \text{lowest value}$
$\bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n} = \frac{\sum_i x_i}{n} = \frac{\sum_i x_i f_i}{n} \quad Z = \frac{X - \bar{x}}{s} \quad \text{margin of error} = \frac{1}{\sqrt{n}} \times 100\%$
$s = \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \dots + (x_n - \bar{x})^2}{n - 1}} = \sqrt{\frac{\sum_i (x_i - \bar{x})^2}{n - 1}} = \sqrt{\frac{\sum_i f_i (x_i - \bar{x})^2}{n - 1}}$

1. (8pts) Pollsters would like to find out how much support there is for raising the minimum wage in a certain city. In order to find out, they consider a survey. Answer whether each of the following methods will produce a good, bad or questionable random sample of voters and **comment why**. Remember you are trying to decide whether every voter has an equal chance of being selected for the sample.

- a  good    Surveying students at the local college.  
 bad  
 iffy
  
- b  good    Surveying random people from the city's property tax records.  
 bad  
 iffy
  
- c  good    Surveying McDonald's patrons.  
 bad  
 iffy
  
- d  good    Surveying church-goers at the city's biggest church.  
 bad  
 iffy

2. (9pts) The starting salaries of a university department's graduates are listed below (in thousands).

- a) Construct a grouped frequency distribution whose first class is 35–39.
- b) Are there more graduates in the top two or the bottom two classes?
- c) Which class has the most graduates?

45, 37, 42, 56, 61, 59, 44, 38, 46, 47,  
 52, 51, 55, 40, 42, 39, 48, 46, 59, 62

Class	Frequency

**3.** (18pts) A homeowner counts the number of eggs he picks up from his chicken coop and records it for two weeks. The numbers he gets are below.

- a) Find the midrange. 7, 1, 3, 0, 5, 9, 4, 7, 3, 8, 4, 3, 3, 6  
b) Find the median.  
c) Find the mean.  
d) Find the range.  
e) Find the standard deviation.

**4.** (25pts) The owners of a fast-food franchise wish to see how quickly they are serving their customers during the 12:00-1:00 hour, so every minute during that hour they count how many cars are waiting in the drive-through lane. The data is below (it shows that 4 times they counted 5 cars waiting, 10 times they counted 6 cars waiting etc.)

- a) Draw a histogram for the data.  
b) Find the mode number of cars waiting.  
c) Find the median number of cars waiting.  
d) Find the mean number of cars waiting.  
e) Find the standard deviation.

Number of cars	Frequency
5	4
6	10
7	16
8	11
9	7
10	12

5. (5pts) Women Ingrid from Norway and Julieta from Spain are 161 cm and 158 cm tall, respectively. The heights of women for their age group are normally distributed with mean 167 cm and standard deviation 2.5 cm for Norway, and mean 164 cm and standard deviation 2.7 cm for Spain. Use  $z$ -scores to determine who is taller relative to other women in their age group in their respective countries.

6. (5pts) A survey of 488 adults found that 17% of them have never had more than \$5,000 in a bank account. Find the margin of error of this survey and explain what it means.

7. (13pts) Weights of produced bags of sugar are normally distributed with mean 1014 grams and standard deviation 7 grams. Use the 68-95-99.7 rule (draw a picture) to find the percentage of bags that weigh

a) between 1007 and 1021 grams

b) under 1000 grams

c) over 1007 grams

d) between 1000 and 1021 grams

8. (17pts) The lifespan of an insect is normally distributed with mean 45 days and standard deviation 6 days. Draw a picture showing which area you are computing as you answer:
- What percentage of these insects lives less than 40 days?
  - What percentage of these insects lives longer than 55 days?
  - What percentage of these insects lives between 30 and 39 days?

**Bonus.** (10pts) Over many years, the organizers of the “Run, Lola, Run” 10K race have found that runners’ times on the race are normally distributed with mean 48 minutes and standard deviation 3 minutes. As the number of participants has risen, the organizers have decided that only runners that have run some other 10K race with a time better than the slowest 15% of “Run, Lola, Run” participants can enter. What is the highest qualifying time for the race? (*Hint: this problem is the inverse of what we usually do: an area is given and we have to find the z-score. Once you have the z-score, the running time can easily be found.*)

**Mathematical Concepts — Exam 4**  
**MAT 117, Fall 2021 — D. Ivanišić**

**Name:** \_\_\_\_\_  
*Show all your work!*

1. (24pts) A city council is choosing which city-improvement project to fund. Improvements to parks, roads or water pipes have been proposed, and councilmembers have ranked them as in the table below.

a) Which choice wins the vote in a plurality election?

b) Which choice wins the vote in a plurality election with a runoff?

c) Which choice is the pairwise comparison winner?

d) Which choice is the winner using Borda's method?

Perform the check on the sum of Borda points.

Votes:	3	2	3	1	4	2
1st	P	P	R	R	W	W
2nd	R	W	P	W	P	R
3rd	W	R	W	P	R	P

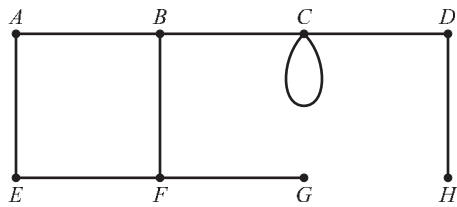
2. (14pts) Members of a bird-watching society are voting for a bird that they plan to put on the society's logo. The choices are: cardinal, duck, heron and woodpecker, and the table shows the percentages of votes that the rankings received.

- a) Which choice wins the vote in a plurality election with elimination?  
 b) Which choice is the winner using Borda's method? Perform the check on the sum of Borda points.

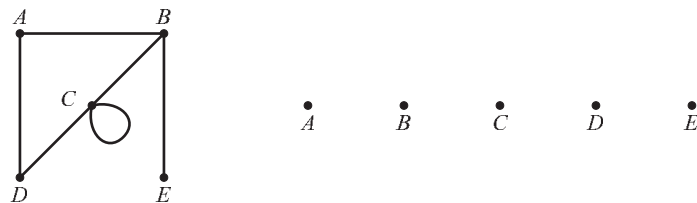
% Votes	21	14	12	20	11	13	9
1st	C	C	D	H	H	W	W
2nd	D	W	H	D	C	D	C
3rd	H	D	W	W	D	H	H
4th	W	H	C	C	W	C	D

3. (12pts) A graph is shown.

- a) Which vertices are adjacent to  $B$ ?  
 b) List the degrees of the vertices.  
 c) List all the bridges on the graph.  
 d) Give any path in which  $B$  is the third vertex.



4. (6pts) A graph is given and a set of vertices. Add edges to the vertices so you get a graph equivalent to the one shown.

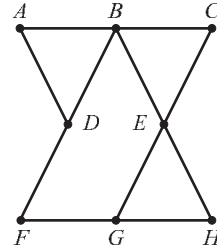
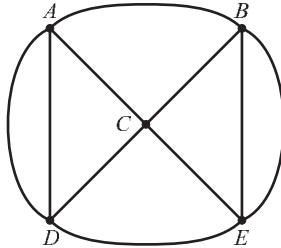
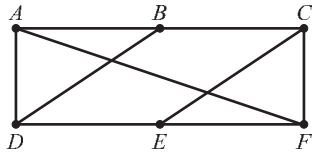


5. (16pts) For each of the following graphs:

a) State and justify whether it has an Euler path.

b) State and justify whether it has an Euler circuit.

c) If it has either an Euler path or a circuit, indicate it on the graph. Use arrows and number the edges to indicate how the Euler path or circuit goes around the graph.

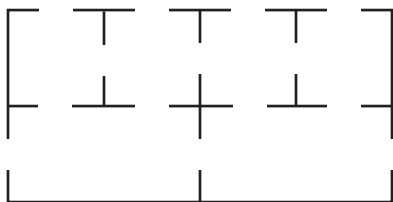


6. (14pts) Below is the floor plan of a house, with doors joining rooms indicated.

a) Represent the floor plan as a graph (rooms are vertices, don't forget an "outside").

b) Use the graph to determine if it is possible to walk around the house, passing through every door exactly once. If it is, draw the route.

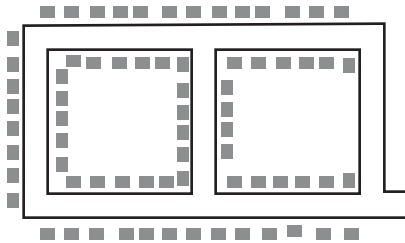
c) Is it possible to do the same as in b), and start and finish in the same room?



7. (14pts) A mail carrier has to deliver mail to the neighborhood shown in the picture. The mail carrier always walks one row of houses on one side of the street at a time.

a) Draw a graph that models the neighborhood.

b) Can the mail carrier deliver the mail to every house in the neighborhood without walking by any row of houses twice and start and end at the same place (for example, on the corner that is the exit of the neighborhood)? If so, display the route.



**Bonus.** (10pts) Draw a graph that has exactly two bridges and whose every vertex has degree 3.



$$I = Prt \quad A = P(1 + rt) \quad A = P \left(1 + \frac{r}{n}\right)^{nt} \quad A = P \frac{\left(1 + \frac{r}{n}\right)^{nt} - 1}{\frac{r}{n}}$$

$$P = PMT \frac{1 - \left(1 + \frac{r}{n}\right)^{-nt}}{\frac{r}{n}} \quad Y = \left(1 + \frac{r}{n}\right)^n - 1$$

$$\frac{a}{b} = \frac{P(E)}{1 - P(E)} \quad P(E) = \frac{a}{a+b} \text{ where odds in favor of } E \text{ are } a : b \quad P(B | A) = \frac{n(A \text{ and } B)}{n(A)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(A \text{ or } B) = P(A) + P(B) \text{ (if } A \text{ and } B \text{ are mutually exclusive)}$$

$$P(A \text{ and } B) = P(A) \cdot P(B | A) \quad P(A \text{ and } B) = P(A) \cdot P(B) \text{ if } A \text{ and } B \text{ are independent}$$

$$E = P_1 \cdot A_1 + P_2 \cdot A_2 + \dots + P_n \cdot A_n$$

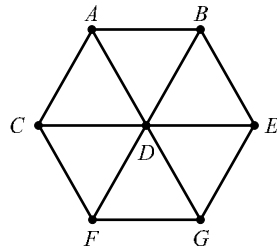
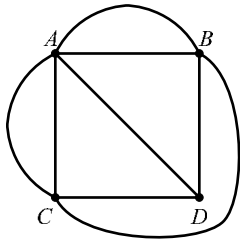
$$\text{midrange} = \frac{\text{lowest value} + \text{highest value}}{2} \quad \text{range} = \text{highest value} - \text{lowest value}$$

$$\bar{x} = \frac{x_1 + x_2 + \dots + x_n}{n} = \frac{\sum_i x_i}{n} = \frac{\sum_i x_i f_i}{n} \quad Z = \frac{X - \bar{x}}{s} \quad \text{margin of error} = \frac{1}{\sqrt{n}} \times 100\%$$

$$s = \sqrt{\frac{(x_1 - \bar{x})^2 + (x_2 - \bar{x})^2 + \dots + (x_n - \bar{x})^2}{n - 1}} = \sqrt{\frac{\sum_i (x_i - \bar{x})^2}{n - 1}} = \sqrt{\frac{\sum_i f_i (x_i - \bar{x})^2}{n - 1}}$$

1. (12pts) Assume the weekly number of hours serious bands practice is normally distributed with a mean of 17 hours and standard deviation 3 hours. Draw pictures showing which area you are computing as you answer:
- What percentage of bands practice less than 13 hours per week?
  - What percentage of bands practice between 15 and 21 hours per week?

2. (12pts) For each of the following graphs:
- State and justify whether it has an Euler path.
  - State and justify whether it has an Euler circuit.
  - If it has either an Euler path or a circuit, indicate it on the graph. Use arrows and number the edges to indicate how the Euler path or circuit goes around the graph.



3. (24pts) Fans of the “Star Wars” saga were asked to elect their favorite episode of the original series (1977-1983). The rankings of the group are below.

- Which choice wins the vote in a plurality election?
  - Which choice wins the vote in a plurality election with elimination?
  - Which choice is the pairwise comparison winner?
  - Which choice is the winner using Borda’s method?
- Perform the check on the sum of Borda points.

Votes:	12	4	11	7	6	3
1st	IV	IV	V	V	VI	VI
2nd	V	VI	IV	VI	IV	V
3rd	VI	V	VI	IV	V	IV

4. (25pts) The age distribution of a class is shown in the table.

- a) Draw a histogram for the data.
- b) Find the mode age.
- c) Find the median age.
- d) Find the mean age.
- e) Find the standard deviation.

Age	Frequency
17	2
18	7
19	13
20	9
21	6
22	3

5. (13pts) Write the probabilities and odds against and in favor of the following events (you can show any work needed below):

Event	probability	odds against	odds in favor
a) Drawing an odd-numbered card from a deck of cards			
b) Getting exactly two heads on three coin tosses			
c) Getting sum 1, 4 or 6 on a roll of two dice			

**6.** (12pts) A spinner has notches numbered 1–9 and is equally likely to stop at any of them. A game of chance is set up like this: the player pays \$5 and spins the spinner. If the numbers 1 or 7 come up, the player wins \$15, if 4 comes up, the player wins \$10, otherwise the player wins nothing.

a) Find the expected value of this game.

b) What is the fair price of this game?

c) If a player played this game 54 times, how much would they expect to win or lose?

**7.** (5pts) In a subdivision of 37 houses, 12 have a pool, 19 have a three-car garage and 4 have both a pool and a three-car garage. If a home is randomly selected from the subdivision, what is the probability that it has a pool or a three-car garage?

**8.** (10pts) The probability that a student gets a job within a year after graduating is 75%. Assuming that different students getting jobs are independent events. What is the probability that:

a) Two students will get jobs after graduating?

b) At least one from a group of three will get a job after graduating?

**9.** (7pts) If \$11,000 is deposited into an account bearing 3.17%, compounded quarterly, how much is in the account after five years?

**10.** (14pts) When her daughter is born, Joanna decides to save \$130,000 to buy her a house or fund her college when she turns 18.

a) How much should she deposit every quarter into an account bearing 6%, compounded monthly?

b) How much of the final amount is from deposits and how much from interest?

**11.** (16pts) True story: physician assistant Hayley Arceneaux spent three days in orbit on a SpaceX spacecraft. Made-up part: as an additional reward, she decided buy a luxury vehicle costing \$65,000, financing it with a 7-year loan at interest rate 1.77%, compounded monthly.

a) What is her montly payment on the loan?

b) What are her total payments over the course of the loan? How much of this amount is for interest?

**Bonus.** (10pts) Over many years, the organizers of the “Run, Lola, Run” 10K race have found that runners’ times on the race are normally distributed with mean 40 minutes and standard deviation 4 minutes. As the number of participants has risen, the organizers have decided that only runners that have run some other 10K race with a time better than the slowest 10% of “Run, Lola, Run” participants can enter. What is the highest qualifying time for the race? (*Hint: this problem is the inverse of what we usually do: an area is given and we have to find the z-score. Once you have the z-score, the running time can easily be found.*)