

Mathematical Concepts — Exam 2
MAT 117, Fall 2012 — D. Ivanišić

Name: _____
Show all your work!

$$\frac{a}{b} = \frac{P(E)}{1-P(E)} \quad P(E) = \frac{a}{a+b} \text{ where odds in favor of } E \text{ are } a : b \quad P(B | A) = \frac{n(A \text{ and } B)}{n(A)}$$

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(A \text{ or } B) = P(A) + P(B) \text{ (if } A \text{ and } B \text{ are mutually exclusive)}$$

$$P(A \text{ and } B) = P(A) \cdot P(B | A) \quad P(A \text{ and } B) = P(A) \cdot P(B) \text{ if } A \text{ and } B \text{ are independent}$$

$$E = P_1 \cdot A_1 + P_2 \cdot A_2 + \cdots + P_n \cdot A_n$$

1. (6pts) In some country license plates of cars consist of three letters and three digits. The middle letter must be a vowel (A, E, I, O, U), while the first letter must be a consonant. The first digit may not be zero or nine, and the three-digit number has to be even. How many different license plates can be made under these rules?

2. (6pts) A student may get a grade of A, B, C, D, E, I in a course. In a class of ten students, how many possibilities are there for the grade sheet of the entire class?

3. (14pts) The table shows the outcomes of car accidents in Florida for a recent year. Write the following probabilities as fractions (that is, no need to write the decimal representation): the probability that a random driver in an accident

- survived?
- did not wear a seat belt?
- wore a seat belt and died?
- died, given that they wore a seat belt?
- survived, given that they did not wear a seat belt?
- wore a seat belt, given that they survived?

| | Wore Seat Belt | No Seat belt | Total |
|-----------------|----------------|--------------|-------|
| Driver Survived | 412,300 | 162,500 | |
| Driver Died | 500 | 1600 | |
| Total | | | |

4. (18pts) Write the probabilities and odds against and in favor of the following events (you can show any work needed below):

| Event | probability | odds against | odds in favor |
|--|-------------|--------------|---------------|
| a) Rolling 1 or 3 on a single roll of a die | | | |
| b) Drawing a queen or a king from a deck of cards | | | |
| c) Getting exactly two heads on three coin tosses | | | |
| d) Getting sum divisible by 5 on a roll of two dice | | | |
| e) One number odd and one even on a roll of two dice | | | |

5. (10pts) A state has 300 numbered highways. Among those, 78 pass by a lake along their route, 123 have a mile-or-more long segment running through a forest, and 36 have both scenic features. If a numbered highway is selected at random, what is the probability it

- passes by a lake along its route or has a mile-or-more segment running through a forest?
- doesn't have either scenic feature?

6. (14pts) Players A and B play the following game: each puts \$1 into a bowl and a die is rolled. If 1 or 5 comes up, A gets \$1.72 from the bowl and B gets the remaining \$0.28. If 2, 3 or 6 come up, A gets \$0.54 from the bowl and B gets the remaining \$1.46. If 4 comes up, both players get \$1 back.

a) Compute the expected value of this game from player A's perspective.

b) Compute the expected value of this game from player B's perspective.

c) Which player would you rather be? How much would your preferred player expect to win if they played 250 games?

7. (14pts) Ambitious Dave discovers that he succeeds in 2 projects out of every 5 he embarks on. Assuming that successes on different projects are independent of one another, what is the probability that, working on

a) three projects, Dave succeeds in all of them?

b) two projects, Dave fails on both?

c) five projects, Dave succeeds on at least one?

8. (18pts) A bargain bin at a bookstore contains 13 crime, 10 romance and 15 fantasy novels. If you pick two books at random from the bin, what is the probability that:
- a) both are fantasy novels?
 - b) neither is a crime novel?
 - c) one is a crime novel and the other is a romance novel?

Bonus. (10pts) Lenka likes to exercise on Wednesday and on Saturday but doesn't always find the time or energy to do it. She exercises on a Wednesday 45% of the time. If she exercises on a Wednesday, then she exercises on the following Saturday 35% of the time. If she doesn't exercise on a Wednesday, then she exercises on the following Saturday 80% of the time. In a random week, what is the probability that she exercises on exactly one of Wednesday or Saturday?