Mathematical Concepts — Final Exam MAT 117, Fall 2021 — D. Ivanšić

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Show all your work!

$I = Prt \quad A = P(1+rt) \quad A = P\left(1+\frac{r}{n}\right)^{nt} \quad A = P\frac{\left(1+\frac{r}{n}\right)^{nt}-1}{\frac{r}{n}}$

$P = PMT\frac{1-\left(1+\frac{r}{n}\right)^{-nt}}{\frac{r}{n}} \quad Y = \left(1+\frac{r}{n}\right)^{n}-1$

$\frac{a}{b} = \frac{P(E)}{1-P(E)} \qquad P(E) = \frac{a}{a+b}$ where odds in favor of E are $a:b$ $\qquad P(B\,|\,A) = \frac{n(A \text{ and } B)}{n(A)}$

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$P(A \text{ or } B) = P(A) + P(B)$ (if A and B are mutually exclusive)

$P(A \text{ and } B) = P(A)\cdot P(B\,|\,A) \qquad P(A \text{ and } B) = P(A)\cdot P(B)$ if A and B are independent

$E = P_1 \cdot A_1 + P_2 \cdot A_2 + \cdots + P_n \cdot A_n$

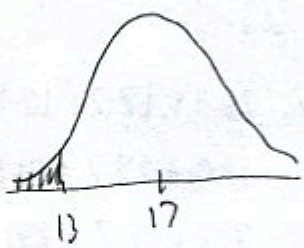
$\text{midrange} = \frac{\text{lowest value} + \text{highest value}}{2} \qquad \text{range} = \text{highest value} - \text{lowest value}$

$\overline{x} = \frac{x_1 + x_2 + \cdots + x_n}{n} = \frac{\sum_i x_i}{n} = \frac{\sum_i x_i f_i}{n} \qquad Z = \frac{X-\overline{x}}{s} \qquad \text{margin of error} = \frac{1}{\sqrt{n}} \times 100\%$

$s = \sqrt{\frac{(x_1-\overline{x})^2 + (x_2-\overline{x})^2 + \cdots + (x_n-\overline{x})^2}{n-1}} = \sqrt{\frac{\sum_i (x_i-\overline{x})^2}{n-1}} = \sqrt{\frac{\sum_i f_i(x_i-\overline{x})^2}{n-1}}$

1. (12pts) Assume the weekly number of hours serious bands practice is normally distributed with a mean of 17 hours and standard deviation 3 hours. Draw pictures showing which area you are computing as you answer:
a) What percentage of bands practice less than 13 hours per week?
b) What percentage of bands practice between 15 and 21 hours per week?

a)



$Z = \frac{13-17}{3} = -\frac{4}{3} \approx -1.33$

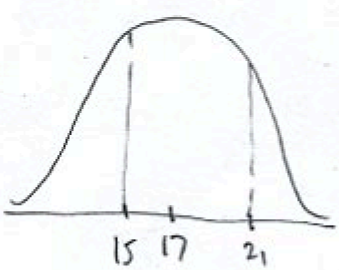
$P(X \le 13)$

$= P(Z \le -1.33)$

$= 0.0918$

9.18%

b)



$Z = \frac{15-17}{3} = -\frac{2}{3} \approx -0.67$

$Z = \frac{21-17}{3} = \frac{4}{3} \approx 1.33$

$P(15 \le X \le 21)$

$= P(-0.67 \le Z \le 1.33)$

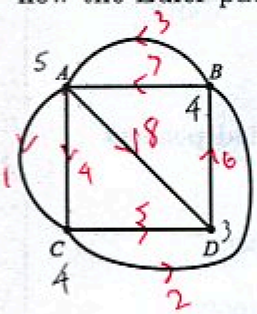
$= A_1 - A_2$

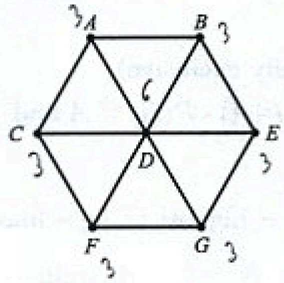
$= 0.9082 - 0.2514$

$= 0.6568$

65.78 %

2. (12pts) For each of the following graphs:
a) State and justify whether it has an Euler path.
b) State and justify whether it has an Euler circuit.
c) If it has either an Euler path or a circuit, indicate it on the graph. Use arrows and number the edges to indicate how the Euler path or circuit goes around the graph.





Left:
Two vertices are odd, so
a) It has an Euler path
b) It does not have an Euler circuit

Right
>2 vertices are odd, so
a) does not have an Euler path
b) does not have an Euler circuit

3. (24pts) Fans of the "Star Wars" saga were asked to elect their favorite episode of the original series (1977-1983). The rankings of the group are below.

a) Which choice wins the vote in a plurality election?
b) Which choice wins the vote in a plurality election with elimination?
c) Which choice is the pairwise comparison winner?
d) Which choice is the winner using Borda's method? Perform the check on the sum of Borda points.

Votes:	12	4	11	7	6	3
1st	IV	IV	V	V	VI	VI
2nd	V	VI	IV	VI	IV	V
3rd	VI	V	VI	IV	V	IV

= 43

a) IV 12+4 = 16
V 11+7 = 18 wins
VI 6+3 = 9

b) IV 16+6 = 22 wins
V 18+3 = 21

c) IV vs V
12+4+6 = 22
11+7+3 = 21

IV vs VI
12+4+11 = 27
6+3+7 = 16

V vs VI
11+7+12 = 30 wins
6+3+4 = 13

Points IV 2 wins
V 1
VI 0

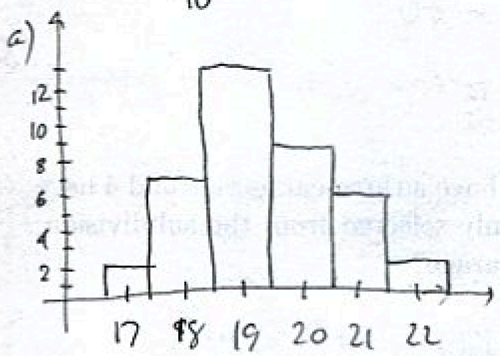
d)
IV: 16·3 + 17·2 + 10·1 = 92
V: 18·3 + 15·2 + 10·1 = 94 wins
VI: 9·3 + 11·2 + 23·1 = 72
258

43 voters × 6 pt per voter
= 258

4. (25pts) The age distribution of a class is shown in the table.
a) Draw a histogram for the data.
b) Find the mode age.
c) Find the median age.
d) Find the mean age.
e) Find the standard deviation.

Age	Frequency
17	2
18	7
19	13
20	9
21	6
22	3
	40

a)



b) mode = 19

c) $\frac{40}{2} = 20$, need 20th & 21st

17, 17, 18, — 18, 19, — 19, 20, — 20, 21, — 21, 22, 22
2nd 9th 22nd

20th & 21st are 19, so median $= \frac{19+19}{2} = 19$

d) $2 \cdot 17 + 7 \cdot 18 + 13 \cdot 19 + 9 \cdot 20 + 6 \cdot 21 + 3 \cdot 22 = 779$

$\bar{x} = \frac{779}{40} = 19.475$

e) $2(17-19.475)^2 + 7(18-19.475)^2 + \dots + 6 \cdot (21-19.475)^2 + 3(22-19.475)^2 = 65.975$

$s = \sqrt{\frac{65.975}{39}} = 1.300641$

5. (13pts) Write the probabilities and odds against and in favor of the following events (you can show any work needed below):

	Event	probability	odds against	odds in favor
a)	Drawing an odd-numbered card from a deck of cards	$\frac{20}{52} = \frac{5}{13}$	8 to 5	5 to 8
b)	Getting exactly two heads on three coin tosses	$\frac{3}{8}$	5 to 3	3 to 5
c)	Getting sum 1, 4 or 6 on a roll of two dice	$\frac{8}{36} = \frac{2}{9}$	7 to 2	2 to 7

a) 1, 3, 5, 7, 9
5 cards × 4 = 20 odd numbered cards in deck

b) HHH
HHT *
HTH *
HTT
THH *
THT
TTH
TTT

exactly two heads here

c) Sum = 1 not possible
4 (1,3), (2,2), (3,1)
6 (1,5), (2,4), (3,3), (4,2), (5,1)
8 outcomes total

6. (12pts) A spinner has notches numbered 1–9 and is equally likely to stop at any of them. A game of chance is set up like this: the player pays \$5 and spins the spinner. If the numbers 1 or 7 come up, the player wins \$15, if 4 comes up, the player wins \$10, otherwise the player wins nothing.

a) Find the expected value of this game.

b) What is the fair price of this game?

c) If a player played this game 54 times, how much would they expect to win or lose?

a)

	outcome	probability
1 or 7	$15-5=10$	$\frac{2}{9}$
4	$10-5=5$	$\frac{1}{9}$
2,3,5,6,8,9	$0-5=-5$	$\frac{6}{9}$

$$E = 10\cdot\frac{2}{9} + 5\cdot\frac{1}{9} + (-5)\cdot\frac{6}{9} = \frac{20+5-30}{9}$$

$$= -\frac{5}{9} \approx 0.56$$

Player can expect to lose on average 0.56 per play

b) $-0.56+5=4.44$

c) $\cancel{54}^{6}\cdot\left(-\frac{5}{\cancel{9}}\right) = -30$

Player expects to lose \$30 when playing 54 times

7. (5pts) In a subdivision of 37 houses, 12 have a pool, 19 have a three-car garage and 4 have both a pool and a three-car garage. If a home is randomly selected from the subdivision, what is the probability that it has a pool or a three-car garage?

$$P(\text{pool or 3-car}) = P(\text{pool}) + P(\text{3-car}) - P(\text{pool AND 3-car})$$

$$= \frac{12}{37} + \frac{19}{37} - \frac{4}{37} = \frac{12+19-4}{37} = \frac{27}{37}$$

8. (10pts) The probability that a student gets a job within a year after graduating is 75%. Assuming that different students getting jobs are independent events. What is the probability that:

a) Two students will get jobs after graduating?

b) At least one from a group of three will get a job after graduating?

a) $P\left(\begin{smallmatrix}\text{student 1}\\ \text{gets job}\end{smallmatrix} \text{ AND } \begin{smallmatrix}\text{student 2}\\ \text{gets job}\end{smallmatrix}\right) = P\left(\begin{smallmatrix}\text{student 1}\\ \text{gets job}\end{smallmatrix}\right)\cdot P\left(\begin{smallmatrix}\text{student 2}\\ \text{gets job}\end{smallmatrix}\right) = 0.75\cdot 0.75 = 0.5625$

b) $P(\text{at least one gets job}) = 1 - P(\text{none get a job}) = 1 - P\left(\begin{smallmatrix}\text{student 1}\\ \text{does not get job}\end{smallmatrix} \text{ AND } \begin{smallmatrix}\text{student 2}\\ \text{does not get job}\end{smallmatrix}\right.$

$\left.\text{AND } \begin{smallmatrix}\text{student 3}\\ \text{does not}\\ \text{get job}\end{smallmatrix}\right) = 1 - 0.25\cdot 0.25\cdot 0.25 = 1 - 0.015625 = 0.984375$

9. (7pts) If \$11,000 is deposited into an account bearing 3.17%, compounded quarterly, how much is in the account after five years?

$$A = P\left(1+\frac{r}{n}\right)^{nt} = 11{,}000\left(1+\frac{0.0317}{4}\right)^{4 \cdot 5} = 11{,}000 \cdot 1.007925^{20}$$

$$= 11{,}000 \cdot 1.171\ldots$$

$$= 12{,}881.22$$

10. (14pts) When her daughter is born, Joanna decides to save \$130,000 to buy her a house or fund her college when she turns 18.

a) How much should she deposit every quarter into an account bearing 6%, compounded ~~monthly~~? quarterly

b) How much of the final amount is from deposits and how much from interest?

a) $$A = P\,\frac{\left(1+\frac{r}{n}\right)^{nt}-1}{\frac{r}{n}}$$

$$130{,}000 = P \cdot \frac{\left(1+\frac{0.06}{4}\right)^{\overset{=72}{4 \cdot 18}}-1}{\frac{0.06}{4}}$$

$$130{,}000 = P \cdot 128.077\ldots$$

$$P = \frac{130{,}000}{128.077} = 1015.02$$

Quarterly deposit of \$1015.02

b) $1015.02 \cdot 72 = 73{,}081.44$ total deposits

$130{,}000 - 73{,}081.44 = 56{,}918.66$ from interest

11. (16pts) True story: physician assistant Hayley Arceneaux spent three days in orbit on a SpaceX spacecraft. Made-up part: as an additional reward, she decided buy a luxury vehicle costing $65,000, financing it with a 7-year loan at interest rate 1.77%, compounded monthly.

a) What is her montly payment on the loan?

b) What are her total payments over the course of the loan? How much of this amount is for interest?

a) $P = PMT \dfrac{1-\left(1+\frac{r}{n}\right)^{-nt}}{\frac{r}{n}}$

84

$65{,}000 = PMT \cdot \dfrac{1-\left(1+\frac{0.0177}{12}\right)^{-12\cdot 7}}{\frac{0.0177}{12}}$

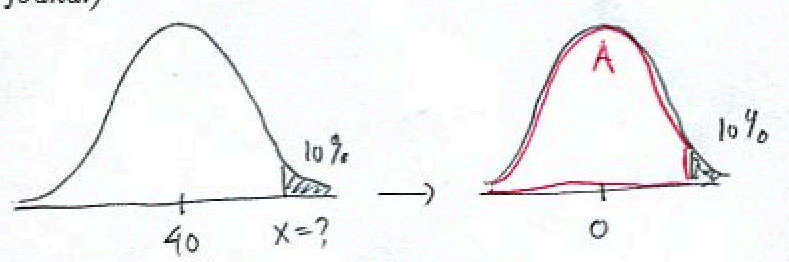
$65{,}000 = PMT \cdot 78.94..$

$PMT = \dfrac{65000}{78.94} = 823.31$

b) $823.31 \cdot 84 = 69{,}158.04$ Total payments

$69{,}158.04 - 65{,}000 = 4158.04$ Total interest

Bonus. (10pts) Over many years, the organizers of the "Run, Lola, Run" 10K race have found that runners' times on the race are normally distributed with mean 40 minutes and standard deviation 4 minutes. As the number of participants has risen, the organizers have decided that only runners that have run some other 10K race with a time better than the slowest 10% of "Run, Lola, Run" participants can enter. What is the highest qualifying time for the race? *(Hint: this problem is the inverse of what we usually do: an area is given and we have to find the z-score. Once you have the z-score, the running time can easily be found.)*



A=0.9, closest value in table to this is 0.8997, for Z=1.28

Runners have to be faster than 1.28 standard deviations above average time.

$40 + 1.28 \cdot 4 = 45.12$

Runners faster than 45.12 minutes qualify for the race.