

1. (24pts) A city council is choosing which city-improvement project to fund. Improvements to parks, roads or water pipes have been proposed, and councilmembers have ranked them as in the table below.

- a) Which choice wins the vote in a plurality election?  
 b) Which choice wins the vote in a plurality election with a runoff?  
 c) Which choice is the pairwise comparison winner?  
 d) Which choice is the winner using Borda's method?  
 Perform the check on the sum of Borda points.

Votes:	3	2	3	1	4	2 = 15 voters
1st	P	P	R	R	W	W
2nd	R	W	P	W	P	R
3rd	W	R	W	P	R	P

a) P  $3+2=5$     b)  $\rightarrow$  P  $5+3=8$  wins  
 R  $3+1=4$   
 W  $4+2=6$  win    W  $6+1=7$

c) P  $3+2+4=9$  w    Points:  
 R  $3+1+2=6$   
 P  $3+2+3=8$  w    P | R | W  
 W  $4+2+1=7$     2 | 0 | 1  
 R  $3+1+3=7$     P wins  
 W  $4+2+2=8$  w    pairwise comparison

d) P  $5 \cdot 3 + 7 \cdot 2 + 3 \cdot 1 = 32$  wins  
 R  $4 \cdot 3 + 5 \cdot 2 + 6 \cdot 1 = 28$   
 W  $6 \cdot 3 + 3 \cdot 2 + 6 \cdot 1 = 30$   
 $\frac{90}{90}$  } equal.  
 15 voters  $\cdot$  6 pts per voter = 90

2. (14pts) Members of a bird-watching society are voting for a bird that they plan to put on the society's logo. The choices are: cardinal, duck, heron and woodpecker, and the table shows the percentages of votes that the rankings received.

- a) Which choice wins the vote in a plurality election with elimination?  
 b) Which choice is the winner using Borda's method? Perform the check on the sum of Borda points.

% Votes	21	14	12	20	11	13	9
1st	C	C	D	H	H	W	W
2nd	D	W	H	D	C	D	C
3rd	H	D	W	W	D	H	H
4th	W	H	C	C	W	C	D

Votes = 100%

1st round

b) 2nd round

3rd round

a) C  $21+14=35 \rightarrow 35$   
 D  $12=12$  *elim*  
 H  $20+11=31 \rightarrow 31+12=43$   
 W  $13+9=22 \rightarrow 22$  *elim*

Heron wins

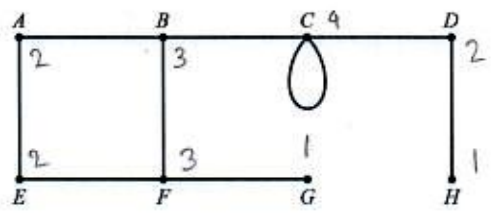
b) C  $35 \cdot 4 + 20 \cdot 3 + 0 \cdot 2 + 45 \cdot 1 = 245$   
 D  $12 \cdot 4 + 54 \cdot 3 + 25 \cdot 2 + 9 \cdot 1 = 269$  *wins*  
 H  $31 \cdot 4 + 12 \cdot 3 + 43 \cdot 2 + 14 \cdot 1 = 260$   
 W  $22 \cdot 4 + 14 \cdot 3 + 32 \cdot 2 + 32 \cdot 1 = 226$

1000

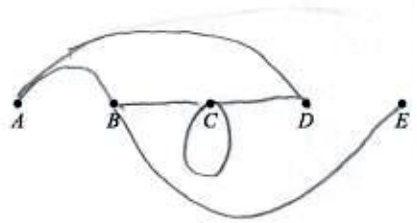
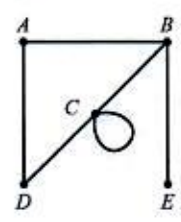
100 votes  $\times$  10 pts per vote = 1000  $\leftarrow$  agrees

3. (12pts) A graph is shown.  
 a) Which vertices are adjacent to B?  
 b) List the degrees of the vertices.  
 c) List all the bridges on the graph.  
 d) Give any path in which B is the third vertex.

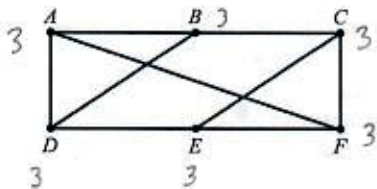
- a) A, F, C  
 b) on picture  
 c) FG, BC, CD, DH



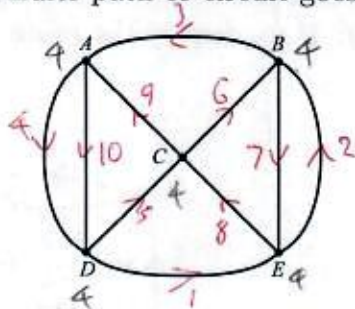
4. (6pts) A graph is given and a set of vertices. Add edges to the vertices so you get a graph equivalent to the one shown.



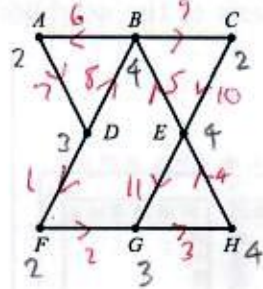
5. (16pts) For each of the following graphs:
- State and justify whether it has an Euler path.
  - State and justify whether it has an Euler circuit.
  - If it has either an Euler path or a circuit, indicate it on the graph. Use arrows and number the edges to indicate how the Euler path or circuit goes around the graph.



Has  $> 2$  odd vertices  
No Euler path nor circuit

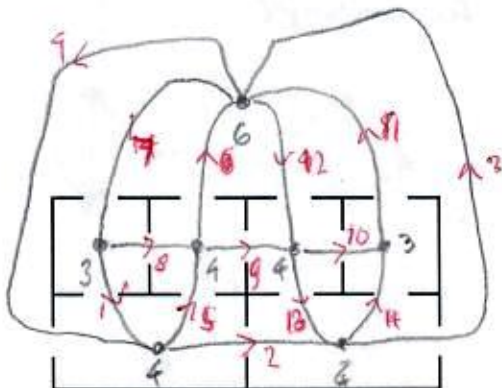


All vertices even  
 $\Rightarrow$  has Euler circuit  
(also has Euler path that starts and ends in same place)



Has exactly 2 odd vertices,  
has Euler path,  
no Euler circuit

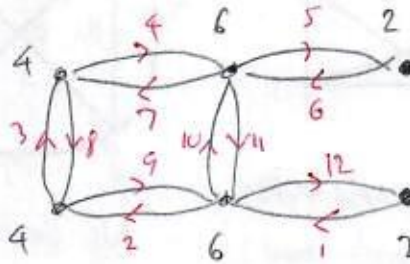
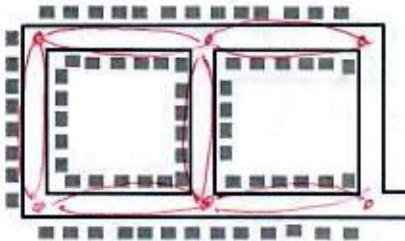
6. (14pts) Below is the floor plan of a house, with doors joining rooms indicated.
- Represent the floor plan as a graph (rooms are vertices, don't forget an "outside").
  - Use the graph to determine if it is possible to walk around the house, passing through every door exactly once. If it is, draw the route.
  - Is it possible to do the same as in b), and start and finish in the same room?



- Graph has exactly two odd vertices, so has an Euler path.
- No, since graph does not have an Euler circuit, as not all vertices are even.

7. (14pts) A mail carrier has to deliver mail to the neighborhood shown in the picture. The mail carrier always walks one row of houses on one side of the street at a time.

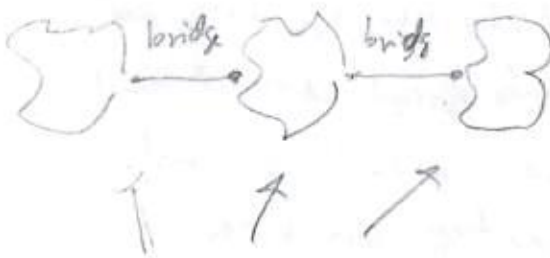
- a) Draw a graph that models the neighborhood.
- b) Can the mail carrier deliver the mail to every house in the neighborhood without walking by any row of houses twice and start and end at the same place (for example, on the corner that is the exit of the neighborhood)? If so, display the route.



- b) Yes, since graph has all vertices even, it has an Euler circuit

**Bonus.** (10pts) Draw a graph that has exactly two bridges and whose every vertex has degree 3.

Start like this



some graph so that  
resulting picture  
has degree of every  
vertex 3

An example

