Mathematical Concepts — Exam 3
MAT 117, Fall 2021 — D. Ivanšić

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Show all your work!

Final answers should have accuracy to 6 decimal places (or 4 decimal places for table-derived answers). Show some work how the mean and standard deviation are computed. *Giving only the answer will bring you few points.*

$$\text{midrange} = \frac{\text{lowest value} + \text{highest value}}{2} \qquad \text{range} = \text{highest value} - \text{lowest value}$$

$$\overline{x} = \frac{x_1 + x_2 + \cdots + x_n}{n} = \frac{\sum_i x_i}{n} = \frac{\sum_i x_i f_i}{n} \qquad Z = \frac{X - \overline{x}}{s} \qquad \text{margin of error} = \frac{1}{\sqrt{n}} \times 100\%$$

$$s = \sqrt{\frac{(x_1 - \overline{x})^2 + (x_2 - \overline{x})^2 + \cdots + (x_n - \overline{x})^2}{n-1}} = \sqrt{\frac{\sum_i (x_i - \overline{x})^2}{n-1}} = \sqrt{\frac{\sum_i f_i (x_i - \overline{x})^2}{n-1}}$$

1. (8pts) Pollsters would like to find out how much support there is for raising the minimum wage in a certain city. In order to find out, they consider a survey. Answer whether each of the following methods will produce a good, bad or questionable random sample of voters and **comment why**. Remember you are trying to decide whether every voter has an equal chance of being selected for the sample.

a ☐ good ☑ bad ☐ iffy — Surveying students at the local college.
Sample has little chance of capturing older people

b ☐ good ☐ bad ☑ iffy — Surveying random people from the city's property tax records.
Not too bad, since many people are property owners, but will miss the poorest part of the population, who are not

c ☐ good ☐ bad ☑ iffy — Surveying McDonald's patrons.
Since almost everyone goes to McD's, you'd get a good sample of the population, but the poor may be represented

d ☐ good ☐ bad ☑ iffy — Surveying church-goers at the city's biggest church.
A large swath of population goes to church, so may get a good sample, unknown if denomination would influence responses

2. (9pts) The starting salaries of a university department's graduates are listed below (in thousands).
a) Construct a grouped frequency distribution whose first class is 35–39.
b) Are there more graduates in the top two or the bottom two clases?
c) Which class has the most graduates?

45, 37, 42, 56, 61, 50, 44, 38, 46, 47,
52, 51, 55, 40, 42, 39, 48, 46, 59, 62

Class	Frequency
35-39	3
40-44	4
45-49	5
50-54	2
55-59	4
60-64	2

$3+4=7$ more graduates in bottom two classes

largest number of graduates in class 45-49

$2+4=6$

3. (18pts) A homeowner counts the number of eggs he picks up from his chicken coop and records it for two weeks. The numbers he gets are below.

7, 1, 3, 0, 5, 9, 4, 7, 3, 8, 4, 3, 3, 6

a) Find the midrange.
b) Find the median. — 14 items
c) Find the mean.
d) Find the range.
e) Find the standard deviation.

0, 1, 3, 3, 3, 3, 4, 4, 5, 6, 7, 7, 8, 9

7th 8th

a) midrange $= \frac{0+9}{2} = 4.5$

b) need 7th, 8th : 4 4 ; $\frac{4+4}{2} = 4$

c) $\frac{0+1+4\cdot3+2\cdot4+5+6+2\cdot7+8+9}{14} = \frac{1+12+8+11+14+17}{14} = \frac{63}{14} = 4.5$

d) range $= 9-0 = 9$

e) $(0-4.5)^2+(1-4.5)^2+4(3-4.5)^2+\dots+(9-4.5)^2 = 89.5$

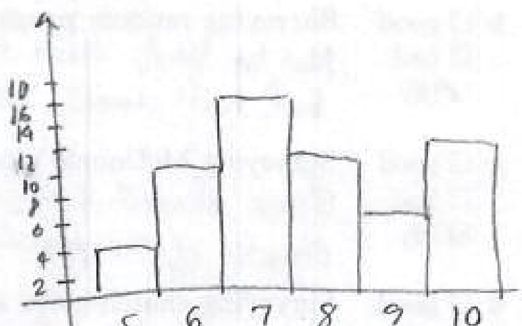
$s = \sqrt{\frac{89.5}{13}} = 2.623855$

4. (25pts) The owners of a fast-food franchise wish to see how quickly they are serving their customers during the 12:00-1:00 hour, so every minute during that hour they count how many cars are waiting in the drive-through lane. The data is below (it shows that 4 times they counted 5 cars waiting, 10 times they counted 6 cars waiting etc.)

a) Draw a histogram for the data.
b) Find the mode number of cars waiting.
c) Find the median number of cars waiting.
d) Find the mean number of cars waiting.
e) Find the standard deviation.

Number of cars	Frequency
5	4
6	10
7	16
8	11
9	7
10	12
	60

a)



b) mode = 7

c) median = 7.5

$5, \dots 5, 6, \dots 6, 7, \dots 7, 8, \dots 8, 9, \dots 9, 10, \dots 10$

4th 14th 30th 31st

$\frac{60}{2} = 30$, need 30th and 31st
7 8

$\frac{7+8}{2} = 7.5$

d) $\frac{4\cdot5+10\cdot6+16\cdot7+11\cdot8+7\cdot9+12\cdot10}{60} = \frac{463}{60}$

$\bar{x} = 7.716667$

e) $4(5-7.71..)^2+10(6-7.71..)^2+\dots+7\cdot(9-7.71..)^2+12(9-7.71..)^2$
$= 142.183..$

$s = \sqrt{\frac{142.183}{60-1}} = 1.552381$

5. (5pts) Women Ingrid from Norway and Julieta from Spain are 161 cm and 158 cm tall, respectively. The heights of women for their age group are normally distributed with mean 167 cm and standard deviation 2.5 cm for Norway, and mean 164 cm and standard deviation 2.7 cm for Spain. Use z-scores to determine who is taller relative to other women in their age group in their respective countries.

$z = \frac{161-167}{2.5} = -2.4$ $\quad$ $z = \frac{158-164}{2.7} = -2.222222$

Ingrid is 2.4 stand dev. below mean

Julieta is 2.22 stand dev. below mean

[Julieta is relatively taller]

6. (5pts) A survey of 488 adults found that 17% of them have never had more than \$5,000 in a bank account. Find the margin of error of this survey and explain what it means.

margin of error is $\frac{1}{\sqrt{488}} \cdot 100\% = 4.526787\% \approx 4.53\%$

Meaning: We can be 95% confident that true percentage is between 17−4.53% and 17+4.53%, i.e. btw 12.47% and 21.53%

7. (13pts) Weights of produced bags of sugar are normally distributed with mean 1014 grams and standard deviation 7 grams. Use the 68-95-99.7 rule (draw a picture) to find the percentage of bags that weigh

a) between 1007 and 1021 grams

68%

b) under 1000 grams

$\frac{95}{2} = 47.5$, $50 - 47.5 = 2.5\%$

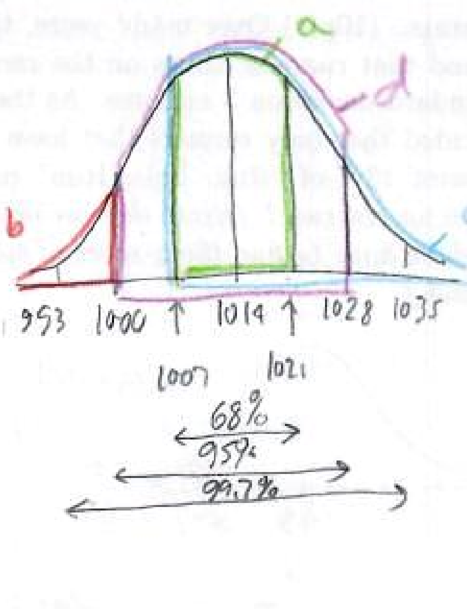
c) over 1007 grams

$\frac{68}{2} = 34$ $\quad$ $50 + 34 = 84\%$

d) between 1000 and 1021 grams

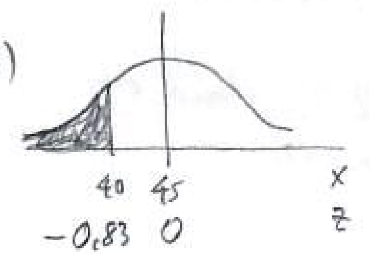
$\frac{95}{2} = 47.5$ $\quad$ $\frac{68}{2} = 34$

$47.5 + 34 = 81.5\%$

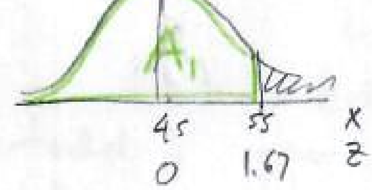


8. (17pts) The lifespan of an insect is normally distributed with mean 45 days and standard deviation 6 days. Draw a picture showing which area you are computing as you answer:
a) What percentage of these insects lives less than 40 days?
b) What percentage of these insects lives longer than 55 days?
c) What percentage of these insects lives between 30 and 39 days?

a)



b)



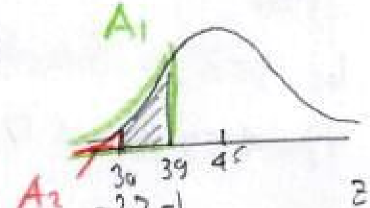
$z = \frac{55-45}{6} = 1.67$

$P(x \le 40) = P(z \le -0.83)$
$= 0.2033, \quad 20.33\%$

$\frac{40-45}{6} = -0.83$

$P(X \ge 40) = P(z \ge 1.67) = 1 - A_1$
$= 1 - 0.9525 = 0.0475, \quad 4.75\%$

d)



$\frac{30-45}{6} = -2.5$

$\frac{39-45}{6} = -1$

$P(30 \le X \le 39) = P(-2.5 \le z \le -1) = A_1 - A_2 = 0.1587 - 0.0062$
$= 0.1525, \quad 15.25\%$

Bonus. (10pts) Over many years, the organizers of the "Run, Lola, Run" 10K race have found that runners' times on the race are normally distributed with mean 48 minutes and standard deviation 3 minutes. As the number of participants has risen, the organizers have decided that only runners that have run some other 10K race with a time better than the slowest 15% of "Run, Lola, Run" participants can enter. What is the highest qualifying time for the race? *(Hint: this problem is the inverse of what we usually do: an area is given and we have to find the z-score. Once you have the z-score, the running time can easily be found.)*

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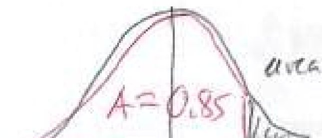

area = 0.85, so need z-score closest to area 1 - 0.15 = 0.85
0.8508 is closest in table, corresponding z-score is 1.04

$X = \bar{X} + z \cdot s = 48 + 1.04 \cdot 3$
$= 51.12$
$= 51$ min 7 sec.

so running time has to be at most 1.04 stand deviations above mean.