Mathematical Concepts — Exam 2 MAT 117, Fall 2021 — D. Ivanšić

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Show all your work!

$\frac{a}{b} = \frac{P(E)}{1-P(E)}$ $P(E) = \frac{a}{a+b}$ where odds in favor of E are $a : b$ $P(B\,|\,A) = \frac{n(A \text{ and } B)}{n(A)}$

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$P(A \text{ or } B) = P(A) + P(B)$ (if A and B are mutually exclusive)

$P(A \text{ and } B) = P(A) \cdot P(B\,|\,A)$ $P(A \text{ and } B) = P(A) \cdot P(B)$ if A and B are independent

$E = P_1 \cdot A_1 + P_2 \cdot A_2 + \cdots + P_n \cdot A_n$

1. (6pts) An exam has 11 multiple-choice questions. The first five have three answers each, and the remaining six have four answers each. If every question is answered with exactly one choice, in how many ways can the exam be answered?

3 for each 4 for each

$3 \times 3 \times \ldots \times 3 \times 4 \times 4 \times \ldots \times 4$

$= 3^5 \cdot 4^6 = 995328$

2. (6pts) An ice cream store has 13 flavors of ice cream, 5 types of cones, 6 varieties of toppings and 4 types of syrup. Assuming you are getting one scoop of ice cream in a cone and the toppings and syrup are optional, how many different ice cream treats can you build?

ic cream	cone	toppings	syrup
13	5	6+1	4+1

(+1 is for omitting option)

$13 \times 5 \times 7 \times 5 = 2275$ options

3. (10pts) The table shows the inventory of a car dealer's lot with respect to age and type. What is the probability, in fraction form, that a random car from this lot:

a) is an SUV?
b) is a new sedan?
c) is an SUV or a pick-up?
d) is a sedan, given it is new?
e) is new, given it is a pick-up?

Type	New	Used	Total
Sedan	5	11	16
SUV	13	25	38
Pick-up	8	6	14
Total	26	42	68

a) $\frac{38}{68} = \frac{19}{34}$ b) $\frac{5}{68}$ c) $\frac{38+14}{68} = \frac{52}{68} = \frac{13}{17}$ d) $\frac{5}{26}$ e) $\frac{8}{14} = \frac{4}{7}$

4. (4pts) Suppose the odds against it raining over a weekend are 5-to-9.
a) What is the probability of it raining? $P(\text{raining}) = \frac{9}{5+9} = \frac{9}{14}$
b) What is the probability of it not raining? $P(\text{not raining}) = \frac{5}{5+9} = \frac{5}{14}$

5. (20pts) Write the probabilities and odds against and in favor of the following events (you can show any work needed below):

	Event	probability	odds against	odds in favor
a)	Getting a 1 or a 4 on a roll of a die	$\frac{2}{6} = \frac{1}{3}$	2 : 1	1 : 2
b)	Drawing a red queen from a deck of cards	$\frac{2}{52} = \frac{1}{26}$	25 : 1	1 : 25
c)	Getting sum 8 on a roll of two dice	$\frac{5}{36}$	31 : 5	5 : 31
d)	Getting the same number on both dice on a roll of two dice	$\frac{6}{36} = \frac{1}{6}$	5 : 1	1 : 5
e)	Getting a tail on the first or third toss of three coin tosses	$\frac{6}{8} = \frac{3}{4}$	1 : 3	3 : 1

c) Sum = 8 for (2,6), (3,5), (4,4), (5,3), (6,2) – 5 outcomes

d) (1,1), (2,2), (3,3), (4,4), (5,5), (6,6) – 6 outcomes

e) HHH THH ✓
HHT ✓ THT ✓ 6 outcomes
HTH TTH ✓
HTT ✓ TTT ✓

6. (12pts) In a collection of draw-by-number apps, 76% are free with ad support and 35% can be purchased ad-free. Suppose all apps in this collection are free with ad support or can be purchased ad-free. If an app is randomly selected from this collection what is the probability it
a) is free with ad support and can be purchased ad-free?
b) lacks at least one of the features above?

a) $\underbrace{P(\text{Free w/ad OR ad-free})}_{\text{equals 1 by 2nd sentence}} = P(\text{Free w/ad}) + P(\text{ad-free}) - P(\text{Free w/ad AND ad-free})$

$1 = 0.76 + 0.35 - P(\text{Free w/ad AND ad-free})$

$1 = 1.11 - P(\text{—})$ $\quad -1.11$

$-0.11 = -P(\text{—})$, so $P(\text{free w/ad AND ad-free}) = 0.11$

b) $P(\text{missing at least one Feature}) = 1 - P(\text{has both}) = 1 - 0.11 = 0.89$

7. (12pts) A game of chance works like this: a player pays \$3 to roll a die. If 1 or 3 comes up, the player wins \$2. If 2, 4 or 5 comes up, the player wins nothing. If 6 comes up, the player wins \$10.
a) Determine the player's expected value.
b) If the player plays this game 60 times, how much do they expect to win or lose?
c) What is the fair price of this game?

a)

	net win	Prob
outcomes	-1	$\frac{2}{6}$
	-3	$\frac{3}{6}$
	7	$\frac{1}{6}$

$$E = (-1)\cdot\frac{2}{6} + (-3)\cdot\frac{3}{6} + 7\cdot\frac{1}{6}$$
$$= \frac{-2-9+7}{6} = -\frac{4}{6} = -\frac{2}{3} \approx -0.67$$

b) $\overset{20}{\cancel{60}}\cdot\left(-\frac{2}{\underset{1}{\cancel{3}}}\right) = -40$ Player expects to lose \$40

c) fair price $= -\frac{2}{3} + 3 = \frac{7}{3} \approx 2.33$

8. (14pts) In these supply-chain burdened days, you can't always find your preferred product at the store. Suppose the probability that you obtain store-brand cream cheese on one visit to a grocery store is 60%. Assume that availability of this cheese on different visits are independent events. What is the probability that:
a) on two trips to the store you find the cheese both times?
b) on three trips to the store, you find the cheese at least once?
c) on three trips to the store, you find the cheese the second and third time, but not the first?

a) $P(\text{C. both times}) = P(\text{C. on 1st AND C. on 2nd})$
$= P(\text{C. on 1st})\cdot P(\text{C. on 2nd}) = 0.6\cdot 0.6 = 0.36$

b) $P(\text{C. on at least one trip}) = 1 - P(\text{no cheese on 3 trips})$
$= 1 - P((\text{no C. on 1st}) \text{ AND } (\text{no C. on 2nd}) \text{ AND } (\text{no C. on 3rd}))$
$= 1 - P(\text{no C. on 1st})\cdot P(\text{no C. on 2nd})\cdot P(\text{no C. on 3rd})$
$= 1 - 0.4\cdot 0.4\cdot 0.4 = 1 - 0.4^3 = 1 - 0.064 = 0.936$

c) $P((\text{no C. on 1st}) \text{ AND } (\text{C. on 2nd}) \text{ AND } (\text{C. on 3rd}))$
$= P(\text{no C. on 1st})\cdot P(\text{C. on 2nd})\cdot P(\text{C. on 3rd}) = 0.4\cdot 0.6\cdot 0.6 = 0.144$

9. (16pts) Two cards are drawn from a deck with 52 cards. What is the probability that
a) The second one is a five, if the first one is a king?
b) The first is a seven, and the second a picture card?
c) Exactly one card is an ace?

a) $P(\text{2nd is } 5 \mid \text{1st is king}) = \frac{4}{51}$

b) $P((\text{1st is } 7) \text{ AND } (\text{2nd is pic})) = P(\text{1st is } 7) \cdot P(\text{2nd is pic} \mid \text{1st is } 7)$

$= \frac{\cancel{4}^{1}}{\cancel{52}_{13}} \cdot \frac{\cancel{12}^{4}}{\cancel{51}_{17}} = \frac{4}{221} = 0.0180995$

c) $P(\text{exactly one is ace}) = P((\text{1st ace AND 2nd not ace}) \text{ OR } (\text{1st not ace AND 2nd ace}))$

$= P(\text{1st ace})P(\text{2nd not ace} \mid \text{1st ace}) + P(\text{1st not ace})P(\text{2nd ace} \mid \text{1st not ace})$

$= \frac{4}{52} \cdot \frac{48}{51} + \frac{48}{52} \cdot \frac{4}{51} = \frac{2 \cdot \cancel{4}^{1} \cdot \cancel{48}^{12}}{\cancel{52}_{13} \cdot \cancel{51}_{17}} = \frac{24}{221} = 0.108597$

Bonus. (10pts) Kids are throwing balls at a wall they made of big foam bricks. Suppose on one throw they hit the wall with probability 0.7. If a single ball hits the wall, the wall collapses with probability 0.2, and if two balls hit the wall, it collapses with probability 0.6. What is the probability that the wall collapses after two throws? *Hint: P(collapse)=P(exactly one ball hits AND collapses)+ P(both balls hit AND collapses)*

$P(\text{collapse}) = P(\text{exactly one ball hits AND collapses}) + P(\text{both balls hit AND collapse})$

$= P(\text{exactly one ball hits}) \cdot P(\text{collapse} \mid \text{exactly one ball hits})$

$+ P(\text{both balls hit}) \cdot P(\text{collapse} \mid \text{both balls hit})$

$= P((\text{1st hits AND 2nd misses}) \text{ OR } (\text{1st misses AND 2nd hits})) \cdot P(\text{collapse} \mid \text{exactly one hits})$

$+ P(\text{1st hits AND 2nd hits}) \cdot P(\text{collapse} \mid \text{both balls hit})$

$= (0.7 \cdot 0.3 + 0.3 \cdot 0.7)\,0.2 + 0.7 \cdot 0.7 \cdot 0.6$

$= 2 \cdot 0.7 \cdot 0.3 \cdot 0.2 + 0.7^2 \cdot 0.6 = 0.084 + 0.294 = 0.378$