Calculus 2 — Exam 0
MAT 308, Spring 2020 — D. Ivanšić

Name: ____________________

Show all your work!

Differentiate and simplify where appropriate:

1. (6pts) $\dfrac{d}{dx}\left(5x^4 - u^3 + \sqrt[4]{x^7} - \dfrac{12}{x^5}\right) =$

2. (6pts) $\dfrac{d}{dt}\,(\sqrt[3]{t} - 1)(\sqrt[3]{t}^{\;2} + \sqrt[3]{t} + 1) =$

3. (8pts) $\dfrac{d}{dz}\,\dfrac{4z^2+1}{(z-5)^2} =$

4. (4pts) $\dfrac{d}{dx}\,\dfrac{1}{xe^x} =$

5. (7pts) (This is a known derivative, your job is to verify it here.)
$\dfrac{d}{d\theta}\,\ln|\sec\theta + \tan\theta| =$

6. (6pts) $\dfrac{d}{dx}\left(\sqrt{x} - \dfrac{3}{\sqrt{x}}\right)\ln x =$

7. (5pts) Let $f(x) = 2^{7x}$. What is $f^{(44)}(x)$, the 44th derivative of f? Justify your answer.

Find the following limits. Use L'Hospital's rule if needed.

8. (2pts) $\displaystyle\lim_{x\to\infty} \frac{1}{x^2} =$

9. (6pts) $\displaystyle\lim_{x\to\infty} \frac{x^3 - 5x^2 + 7x + 9}{x^2 - 4x + 5} =$

10. (8pts) $\displaystyle\lim_{x\to 0} (1 + 2x)^{\frac{1}{x}} =$

Find the following antiderivatives.

11. (7pts) $\displaystyle\int 4x^4 - \frac{4}{\sqrt{1-x^2}} + \sqrt[3]{x^{11}} + e^3\,dx =$

12. (3pts) $\displaystyle\int \cos(4x+1)\,dx =$

13. (7pts) $\displaystyle\int \frac{x^2+1}{\sqrt{x}}\,dx =$

Use the substitution rule in the following integrals:

14. (7pts) $\displaystyle\int \frac{4x-3}{(4x^2-6x+5)^3}\,dx =$

15. (10pts) $\displaystyle\int_0^{\frac{2\pi}{3}} \frac{\sin x}{1+\cos^2 x}\,dx =$

16. (8pts) Find the equation of the tangent line to the curve $y = x^2 + 2x - 15$ at the point $(2, -7)$. Sketch the curve and the tangent line on the same graph.

Bonus. (10pts) The rear inside cover of our book claims that

$$\int x^2 \sqrt{a^2 - x^2}\, dx = \frac{x}{8}(2x^2 - a^2)\sqrt{a^2 - x^2} + \frac{a^4}{8} \arcsin \frac{x}{a} + C$$

Verify this formula by differentiating.

Calculus 2 — Exam 1 **MAT 308, Spring 2020 — D. Ivanšić**

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Show all your work!

Find the following integrals:

1. (6pts) $\displaystyle\int x\cos(3x)\,dx =$

2. (9pts) $\displaystyle\int \sin^3 x\cos^3 x\,dx =$

Determine whether the following improper integrals converge, and, if so, evaluate them.

3. (8pts) $\displaystyle\int_1^\infty \frac{x+1}{x^3}\,dx =$

4. (8pts) $\displaystyle\int_0^\infty \frac{1}{1+x^2}\,dx =$

Use trigonometric substitution to evaluate the following integrals. Don't forget to return to the original variable where appropriate.

5. (12pts) $\displaystyle\int \frac{x^3}{\sqrt{x^2+7}}\,dx =$

6. (14pts) $\displaystyle\int_0^{\sqrt{2}} x^2\sqrt{4-x^2}\,dx =$

Use the method of partial fractions to find the following integrals.

7. (14pts) $\displaystyle\int \frac{3x^3 - 5x^2 + 9x - 5}{(x^2+1)^2}\, dx =$

8. (9pts) Use comparison to determine whether the improper integral $\displaystyle\int_0^{\frac{\pi}{4}} \frac{\cos x}{x}\, dx$ converges.

9. (20pts) The integral $\int_0^3 e^{-x^2}\,dx$ is given. It cannot be found by antidifferentiation, since the antiderivative of e^{-x^2} is not expressible using elementary functions.

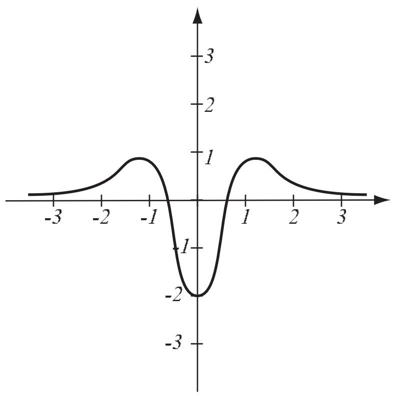
a) Write the expression you would use to calculate M_6, the midpoint rule with 6 subintervals. All the terms need to be explicitly written, do not use f in the sum.

b) Find y'' for $y = e^{-x^2}$.

c) The graph of y'' is shown: use it to find the error estimate for M_n in general.

d) Estimate the error for M_6.

e) What should n be in order for M_n to give you an error less than 10^{-4}?



Bonus (10pts) Find the reduction formula that reduces $\displaystyle\int \frac{dx}{(x^2+a^2)^n}$ to $\displaystyle\int \frac{dx}{(x^2+a^2)^{n-1}}$.

Start on $\displaystyle\int \frac{dx}{(x^2+a^2)^{n-1}}$ with an integration by parts, then rewrite an x^2 in the new integral as $x^2 + a^2 - a^2$ and see what you can do.

Calculus 2 — Exam 2
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Name: ______________________________

Show all your work!

1. (24pts) The region bounded by the curves $y = x^2$ and $y = 2 - x$ is rotated around the x-axis.
a) Sketch the solid and a typical cross-sectional washer.
b) Set up the integral for the volume of the solid.
c) Evaluate the integral.

2. (14pts) Consider the triangle bounded by lines $y = \frac{1}{2}x + 1$, $y = -x$ and $y = 2$.
a) Sketch the triangle.
b) Set up the integral that computes its area. Simplify, but do not evaluate the integral.

3. (16pts) There are infinitely many regions that are above line $y = \frac{1}{2}$ and below the curve $y = \cos x$. Rotate the region that intersects the y-axis about the y-axis to get a solid.
a) Sketch the solid and a typical cylindrical shell.
b) Set up the integral for the volume of the solid using the shell method. Simplify, but do not evaluate the integral.

4. (16pts) The base of a solid is the triangle in the xy-plane with vertices $A = (0, 0)$, $B = (2, 0)$ and $C = (0, 4)$. The cross-sections of the solid perpendicular to the x-axis are half-disks whose diameters lie in the triangle.
a) Sketch the solid and a typical cross-section.
b) Set up the integral for the volume of the solid. Simplify, but do not evaluate the integral.

5. (14pts) Compute the length of the curve $y = \frac{2}{3}x^{\frac{3}{2}} - \frac{1}{2}x^{\frac{1}{2}}$ from $x = 1$ to $x = 4$.

6. (16pts) A leaky bucket is lifted from a well with depth 20 meters to the surface. The bucket weighs 1kg, starts with 10 liters of water at bottom and has only 2 liters by the time it is pulled to the top (assume it empties at a constant rate and rope weight is negligible). Set up the integral for the work needed to lift the bucket from the bottom of the well to the top. Assume $g = 10$ and water density = 1kg/liter. Simplify, but do not evaluate the integral.

Bonus (10pts) Consider the surface obtained by rotating the curve $y = e^x$, $-1 \leq x \leq 1$, around the x-axis.
a) Set up the integral for surface area in variable x.
b) Set up the integral for surface area in variable y.
c) Do not evaluate the integrals, but verify that they are equal.

Calculus 2 — Exam 3 **MAT 308, Spring 2020 — D. Ivanšić**

Name: ____________________

Show all your work!

Find the limits, if they exist.

1. (6pts) $\lim_{n\to\infty} \frac{4^{n-1}}{3^{2n}} =$

2. (6pts) $\lim_{n\to\infty} \cos \frac{n\pi}{2} =$

3. (10pts) Find the limit. Use the theorem that rhymes with what a person might do, if an irritant enters their nose.

$$\lim_{n\to\infty} \frac{\sin n + 2\cos n}{3n - 7}$$

4. (6pts) Write the series using summation notation:

$$\frac{9}{1} - \frac{27}{1 \cdot 2} + \frac{81}{1 \cdot 2 \cdot 3 \cdot 4} - \frac{243}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6} + \cdots =$$

5. (12pts) Justify why the series converges and find its sum.

$$\sum_{n=2}^{\infty} (-1)^n \frac{5^{n-1}}{3^{2n+1}} =$$

Determine whether the following series converge and justify your answer.

6. (6pts) $\sum_{n=1}^{\infty} e^{\frac{1}{n}}$

7. (12pts) $\sum_{n=1}^{\infty} \frac{\sqrt{n}+3}{n^2 - 2n - 3}$

8. (22pts) Consider the alternating series $\sum_{n=2}^{\infty}(-1)^n \frac{n^2}{n^3+7}$.

a) Show that the sequence $\frac{n^2}{n^3+7}$ is decreasing from some point on.
b) Show the limit of the sequence in a) is 0.
c) Is the series convergent?
d) Is the series absolutely convergent? Use the integral test.

Determine whether the following series converge using the root or ratio test.

9. (10pts) $\sum_{n=3}^{\infty} \frac{5^{3n}}{(n+1)!}$

10. (10pts) $\displaystyle\sum_{n=1}^{\infty}(-1)^n \frac{(\arctan n)^n}{n^2+4n}$

Bonus. (10pts) Does $\displaystyle\sum_{n=0}^{\infty}\frac{2^n+3^n}{4^n+5^n}$ converge? (Hint: root test and dominant terms.)

Calculus 2 — Exam 4 **MAT 308, Spring 2020 — D. Ivanšić**

Name: ______________________________

Show all your work!

If you are filming yourself as you take the exam for later upload, write **code: G9LK6A** on the first sheet of paper with your solutions. Then hold the paper at the beginning so the code can be captured by the camera.

Find the intervals of convergence for the series below. Don't forget to check the endpoints.

1. (16pts) $\displaystyle\sum_{n=0}^{\infty} \frac{n^2+7n}{5^n}(x-3)^n.$

2. (10pts) $\displaystyle\sum_{n=1}^{\infty} \frac{4^n}{1\cdot 3\cdot 5\cdot \cdots \cdot (2n-1)}(x+1)^n.$

3. (6pts) Use a known power series to find the sum:

$$\sum_{n=0}^{\infty} \frac{(-1)^n}{2^n \cdot n!} =$$

4. (8pts) Use a known power series to find the limit.

$$\lim_{x \to 0} \frac{\sin x - x + \frac{x^3}{6}}{x^5} =$$

5. (14pts) Use the geometric series to get a power series for $\frac{1}{(1-3x)^2}$. State the interval of convergence (no need to check the endpoints).

6. (14pts) Use the binomial series (expand the binomial coefficients, and simplify) to write the power series expansion of the function.

$\sqrt[4]{1+x} =$

7. (18pts) Let $f(x) = \cos x$.
a) Find the 3rd Taylor polynomial for f centered at $a = \dfrac{\pi}{4}$.
b) Use Taylor's formula to get an estimate of the error $|R_3|$ on the interval $\left(\dfrac{\pi}{8}, \dfrac{3\pi}{8}\right)$. Leave your answer as a fraction.

8. (14pts) Use the known power series for $\ln(1+x)$ to give an estimate of $\ln 1.5$ with accuracy 10^{-2}. Write the estimate as a sum (you do not have to simplify it).

Bonus (10pts) Find the Maclaurin series for $\arcsin x$. (Hint: what is the derivative of this function?)

Calculus 2 — Exam 5 **MAT 308, Spring 2020 — D. Ivanšić**	**Name:** ______________________ *Show all your work!*

If you are filming yourself as you take the exam for later upload, write **code: A78BP3** on the first sheet of paper with your solutions. Then hold the paper at the beginning so the code can be captured by the camera.

1. (12pts) Polar coordinates of two points are given.
a) Sketch the points in the plane.
b) For each point, give two additional polar coordinates, one with a positive r, one with a negative r.

$\left(3, -\frac{\pi}{3}\right)$ $\qquad\qquad$ $\left(-2, \frac{4\pi}{5}\right)$

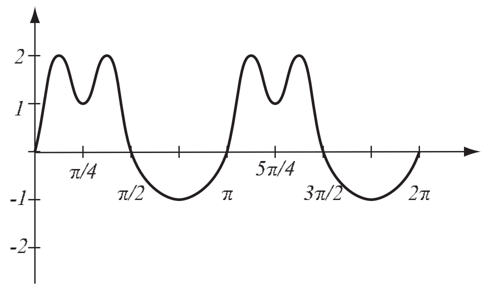
2. (10pts) Convert (a picture may help):
a) $\left(7, -\frac{\pi}{4}\right)$ from polar to rectangular coordinates
b) $(-2, 2\sqrt{3})$ from rectangular to polar coordinates

3. (12pts) Find the equation of the tangent line to the parametric curve $x = t^3 - 4t^2$, $y = t^3 + 5t$ at the point when $t = 1$.

4. (12pts) A particle moves along the path with parametric equations $x(t) = \cos t$, $y(t) = -\sin^2 t$ for $-\pi \leq t \leq \pi$. Eliminate the parameter in order to sketch the path of motion and then describe the motion of the particle.

5. (8pts) Find the length of the parametric curve given by $x(t) = 1 + 2t$, $y(t) = 3 - t$, $0 \leq t \leq 1$.

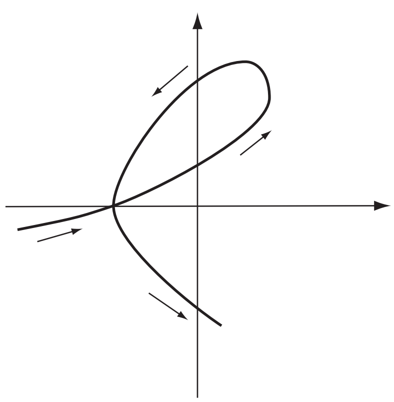
6. (12pts) The graph of $r = f(\theta)$ is given in cartesian coordinates. Use its intervals of increase and decrease to help you sketch the polar curve $r = f(\theta)$. Indicate which piece of the cartesian graph corresponds to which piece of the polar graph.



7. (22pts) Find the area inside the curve $r = 1$ and outside $r = 2(1 - \sin\theta)$. Draw a picture showing the area you are computing.

8. (12pts) In a misguided attempt to fight the coronavirus, a bottle of disinfectant is thrown from the origin so that its position is given by $x(t) = 20t$, $y(t) = 18t - 5t^2$, where length is measured in meters, time in seconds.
a) Kshhhh! When does the bottle hit the ground?
b) How far did it travel from the origin?

Bonus. (10pts) The graph of the parametric curve $x(t) = t^3 - 12t$, $y(t) = -t^2 - 2t + 8$ is shown. Set up the integral(s) needed to find the area enclosed by the loop.



Calculus 2 — Final Exam
MAT 308, Spring 2020 — D. Ivanšić

Name: ____________________

Show all your work!

If you are filming yourself as you take the exam for later upload, write **code: 32FE5Y** on the first sheet of paper with your solutions. Then hold the paper at the beginning so the code can be captured by the camera.

Find the following integrals:

1. (6pts) $\int xe^{2x}\,dx =$

2. (10pts) $\int \sec^4 x \tan^3 x\,dx =$

3. (12pts) Use trigonometric substitution to evaluate the integral.

$$\int \frac{x^3}{\sqrt{9-x^2}}\,dx =$$

4. (6pts) Determine whether the following improper integral converges, and, if so, evaluate it.

$$\int_0^\infty \frac{1}{1+x^2}\,dx =$$

5. (16pts) The region bounded by the curves $y = x^2 - 4x$ and $y = 30 - x^2$ is rotated around the x-axis.

a) Sketch the solid and a typical cross-sectional washer.

b) Set up the integral for the volume of the solid. Simplify, but do not evaluate the integral.

6. (10pts) Justify why the series converges and find its sum.

$$\sum_{n=0}^{\infty} \frac{2^{2n+1}}{7^{n-1}} =$$

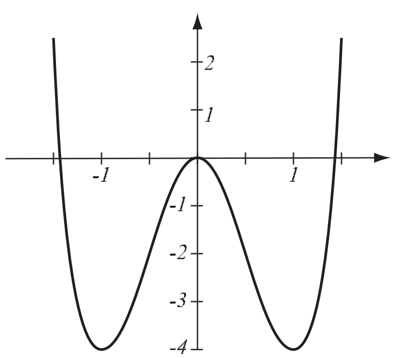
7. (14pts) Find the interval of convergence of the series. Don't forget to check the endpoints.

$$\sum_{n=0}^{\infty} \frac{(x-3)^n}{2^{n+1}(n+4)}.$$

8. (16pts) Let $f(x) = \ln x$.
a) Find the 3rd Taylor polynomial for f centered at $a = 10$.
b) Use Taylor's formula to get an estimate of the error $|R_3|$ on the interval $(8, 12)$.

9. (10pts) A particle moves along the path with parametric equations $x(t) = \cos t$, $y(t) = 4 + \sin^2 t$, $0 \leq t \leq 2\pi$. Eliminate the parameter in order to sketch the path of motion and then describe the motion of the particle.

10. (24pts) The integral $\int_0^1 \cos(x^2)\,dx$ is given. It cannot be found by antidifferentiation, since the antiderivative of $\cos(x^2)$ is not expressible using elementary functions.
a) Write the expression you would use to calculate M_6, the midpoint rule with 6 subintervals. All the terms need to be explicitly written, do not use f in the sum.
b) The graph of y'' is shown: use it to find the error estimate for M_n in general.
c) What should n be in order for M_n to give you an error less than 10^{-4}?
d) Use a known power series for to find a power series for the above integral.
e) How many terms of the power series are needed to estimate the integral to accuracy 10^{-4}? Write the estimate as a sum (you do not have to simplify it).
f) Which method requires less computation to evaluate the integral with accuracy 10^{-4}, midpoint formula or series?



11. (10pts) In another attempt to fight the coronavirus, a bottle of disinfectant is thrown from the origin so that its position is given by $x(t) = 15t$, $y(t) = 16t - 5t^2$, where length is measured in meters, time in seconds. Find the equation of the tangent line to this curve when $t = 2$.

12. (16pts) Find the area inside the polar curve $r = 2\cos\theta$ and outside $r = 1$. Draw a picture showing the area you are computing.

Bonus. (15pts) The graph of the parametric curve $x(t) = t^3 - 12t$, $y(t) = -t^2 - 2t + 8$ is shown. Compute the area enclosed by the loop.