Calculus 2 — Final Exam **MAT 308, Fall 2011 — D. Ivanšić**

Name: ______________________________

Show all your work!

1. (12pts) Consider the region enclosed by the curves $y = x^2 - 2x - 3$ and $y = x + 7$.
a) Sketch the region.
b) Set up the integral that computes its area. Do not evaluate the integral.

2. (16pts) Consider the region bounded by the curves $y = \sin x$ and $y = x$ and $x = \frac{\pi}{2}$. Recall that $\sin x < x$ for $x > 0$.
a) Find the volume of the solid obtained by rotating this region about the x-axis.
b) Sketch the solid and its typical cross-section.

3. (12pts) Integrate: $\displaystyle\int_{-\frac{\pi}{6}}^{\frac{\pi}{6}} \sin^2 x \cos^3 x \, dx =$

4. (12pts) Use trigonometric substitution to evaluate: $\displaystyle\int \frac{x^2}{x^2+9} \, dx =$

5. (16pts) The integral $\int_0^1 e^{x^2} \, dx$ is given. It cannot be found by antidifferentiation, since the antiderivative of e^{x^2} is not expressible using elementary functions.

a) Write out the expression for M_4 for this particular example, the midpoint rule with 4 subintervals.

b) Use the error estimate $|\text{Error}(M_n)| \leq \frac{K_2(b-a)^3}{24n^2}$ to determine how many subintervals are needed if we wish that M_n gives us an error less than 10^{-3}?

Determine whether the following improper integrals converge, and, if so, evaluate them.

6. (6pts) $\displaystyle\int_2^\infty \frac{1}{\sqrt[5]{x}}\,dx =$

7. (10pts) $\displaystyle\int_0^\infty xe^{-x}\,dx =$

8. (8pts) Determine whether the series $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+4n}{n^5+2n^3}$ converges.

9. (10pts) Justify why the series converges and find its sum.

$$\sum_{n=1}^{\infty} \frac{7 \cdot 3^{n+1}}{2^{2n-1}} =$$

10. (14pts) Find the interval of convergence for the series $\sum_{n=1}^{\infty} n(x-2)^n$. Don't forget to check the endpoints of the interval for convergence.

11. (16pts) a) Write the series expansion for $\dfrac{1}{1+x}$ and state where it converges.
b) Integrate both sides of the equation in a) from $x = 0$ to $x = \frac{1}{2}$.
c) How many terms of the series on the right side of b) would be needed to compute $\ln \frac{3}{2}$ with accuracy 10^{-2}? Write the corresponding partial sum and simplify it to a fraction. (Recall the error estimate: $|s - s_n| < a_{n+1}$.)

12. (8pts) Convert (a picture may help):
a) $\left(2\sqrt{2}, \dfrac{5\pi}{4}\right)$ from polar to rectangular coordinates
b) $(-3, \sqrt{3})$ from rectangular to polar coordinates

13. (10pts) A particle moves along the path $c(t) = (3 + 2\sin t, -2 - 2\cos t)$, for $0 \le t \le \pi$. Eliminate the parameter in order to sketch the path of motion and then describe the motion of the particle.

Bonus. (8pts) Determine whether the series converges.

$$\sum_{n=1}^{\infty} \left(\frac{n}{n+1} \right)^{n^2}$$

Bonus. (7pts) A particle is traveling along the polar curve $r = f(\theta)$, $\theta \ge 0$, where we also treat θ as time. Find the general expression for the speed of the particle at time θ. *(Hint: you would know how to do this problem if you had parametric equations for x and y, wouldn't you?)*