Calculus 2 — Exam 6
MAT 308, Fall 2011 — D. Ivanšić

Name: ____________________

Show all your work!

Determine whether the following series converge:

1. (10pts) $\displaystyle\sum_{n=1}^{\infty} \frac{n^3+4n+1}{3^{n+4}}$

2. (10pts) $\displaystyle\sum_{n=3}^{\infty} \frac{4^{3n+1}}{n!}$

3. (10pts) Write $\dfrac{5}{7+3x}$ as a power series and indicate the interval where this expansion is valid (do not check the endpoints of the interval for convergence).

4. (20pts) Find the interval of convergence for the series $\displaystyle\sum_{n=1}^{\infty} \frac{(x-3)^n}{5^n(n^4+n^2+1)}$. Don't forget to check the endpoints of the interval for convergence.

5. (12pts) Use the alternating series test to show that the series $\displaystyle\sum_{n=1}^{\infty} (-1)^n \frac{n}{n^2+15}$ converges. Thoroughly check the conditions of the test.

6. (8pts) Use the Maclaurin series for e^x to show that $\dfrac{d}{dx}\, e^x = e^x$.

7. (16pts) The integral $\int_0^1 \sin x^2\, dx$ cannot be found by antidifferentiation, since the antiderivative of $\sin x^2$ is not expressible using elementary functions. However, we can estimate it using series as follows:
a) Use the known Maclaurin series for $\sin x$ to write the Maclaurin series for $\sin x^2$.
b) Integrate the series to find $\int_0^1 \sin x^2\, dx$, represented as a series.
c) How many terms of the series in b) would be needed to approximate the integral with accuracy 10^{-3}? Write the corresponding partial sum and simplify it to a fraction. (Recall the error estimate: $|s - s_n| < a_{n+1}$.)

8. (14pts) Find the Taylor series expansion of $f(x) = \dfrac{1}{x}$ about $a = 2$. (Use the the general formula for a Taylor series. Or, apply a tricked-out geometric series.)

Bonus. (10pts) Determine whether the series converges.

$$\sum_{n=1}^{\infty} \left(\frac{n}{n+1} \right)^{n^2}$$