Calculus 2 — Exam 5
MAT 308, Fall 2011 — D. Ivanšić

Name: ____________________

Show all your work!

Find the limits, if they exist.

1. (4pts) $\lim_{n\to\infty} \dfrac{100^n}{n!} =$

2. (8pts) $\lim_{n\to\infty} \sin \dfrac{(2n-1)\pi}{4} =$

3. (12pts) $\lim_{n\to\infty} \left(1 - \dfrac{1}{n}\right)^n =$

4. (10pts) Use the theorem that rhymes with the tool that unlocks doors to find:

$\lim_{n\to\infty} \dfrac{2+\sin n}{n^2+4}$

5. (6pts) Write the series using summation notation:

$$\frac{4}{1} - \frac{8}{1 \cdot 2} + \frac{16}{1 \cdot 2 \cdot 3} - \frac{32}{1 \cdot 2 \cdot 3 \cdot 4} + \cdots =$$

6. (12pts) Justify why the series converges and find its sum.

$$\sum_{n=3}^{\infty} \frac{5 \cdot 2^{n-1}}{3^{n+1}} =$$

Determine whether the following series converge and justify your answer.

7. (6pts) $\displaystyle\sum_{n=1}^{\infty} \frac{n^2+3}{n^2-1}$

8. (12pts) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n^2+3n}}{n^3-4n^2+1}$

9. (12pts) $\displaystyle\sum_{n=1}^{\infty} \frac{3^n - n^3}{n^4 + 4^n}$

10. (18pts) Consider the series $\displaystyle\sum_{n=1}^{\infty} ne^{-n}$.

a) Show that $f(x) = xe^{-x}$ is decreasing from some point on and positive for $x \geq 0$.

b) Justify why you may use the integral test on this series and apply it to determine whether the series converges. *(Hint: use integration by parts.)*

Bonus. (10pts) For which $p > 0$ does $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^p \ln n}$ converge?