Calculus 2 — Exam 4
MAT 308, Fall 2011 — D. Ivanšić

Name: ______________________________

Show all your work!

Determine whether the following improper integrals converge, and, if so, evaluate them.

1. (8pts) $\displaystyle\int_5^\infty \frac{1}{x^{\frac{7}{9}}}\,dx =$

2. (10pts) $\displaystyle\int_0^\infty e^{-3x}\,dx =$

3. (14pts) $\displaystyle\int_2^\infty \frac{1}{x(\ln x)^2}\,dx =$

4. (12pts) Use comparison to determine whether the improper integral $\displaystyle\int_0^1 \frac{e^x}{\sqrt{x}}$ converges.

5. (14pts) Find the length of the curve $y = \dfrac{x^5}{5} + \dfrac{1}{12x^3}$ from $x = 1$ to $x = 3$.

6. (20pts) The integral $\int_0^{1.2} \sin x^2\, dx$ is given. It cannot be found by antidifferentiation, since the antiderivative of $\sin x^2$ is not expressible using elementary functions.

a) Use a program to find M_{25}, the midpoint rule with 25 subintervals.

b) Use the error estimate $|\text{Error}(M_n)| \leq \frac{K_2(b-a)^3}{24n^2}$ to estimate the error that the midpoint rule makes for $n = 25$.

c) What should n be in order for M_n to give you an error less than 10^{-6}?

7. (22pts) Let $f(x) = \sqrt{x}$.
a) Find the 3rd Taylor polynomial for f centered at $a = 4$.
b) Use your calculator to compute the error $|\sqrt{4.5} - T_3(4.5)|$.
c) Use the error estimate $|f(x) - T_n(x)| \leq K_{n+1}\frac{|x-a|^{n+1}}{(n+1)!}$ to estimate the error $|\sqrt{4.5} - T_3(4.5)|$. Does the actual error satisfy this error estimate?
d) How big should n be if we wish to estimate $\sqrt{4.5}$ using $T_n(4.5)$ with accuracy 10^{-7}?

Bonus (10pts) Redoing problem 6, estimate $\int_0^{1.2} \sin x^2\, dx$ using the Maclaurin polynomial $T_n(x)$ for $\sin x$ by following these steps.
a) Find an n such that $|\sin x - T_n(x)| \le \frac{5}{6} \times 10^{-6}$ on the entire interval $[0, 1.44]$.
b) Write $T_n(x)$ for the n you found in a).
c) Find the exact value of $\int_0^{1.2} T_n(x^2)\, dx$ and give its decimal value. (Note: you are integrating $T_n(x^2)$, not $T_n(x)$.)
d) Theory guarantees that your answer in c) approximates $\int_0^{1.2} \sin x^2\, dx$ with accuracy 10^{-6}. From a machine's point of view, which of the two approaches required fewer calculations? (Note: $\frac{5}{6}$ and 1.44 appear in a) because $\frac{5}{6} = \frac{1}{1.2-0}$ and $1.44 = 1.2^2$.)