

**Calculus 2 — Exam 1**  
**MAT 308, Fall 2011 — D. Ivanišić**

**Name:** \_\_\_\_\_  
*Show all your work!*

Differentiate and simplify where appropriate:

1. (6pts)  $\frac{d}{dt} (\sqrt{t} + \sqrt[3]{t})(\sqrt{t} - \sqrt[3]{t}) =$

2. (5pts)  $\frac{d}{dx} (5x^2 - 4x)e^x =$

3. (8pts)  $\frac{d}{dz} \frac{z^2 + \sqrt{z}}{z^2 - \sqrt{z}} =$

4. (4pts)  $\frac{d}{dx} \frac{1}{x^2 - 3x + 1} =$

5. (8pts)  $\frac{d}{d\theta} \frac{\cos \theta}{\sin^2 \theta} =$

6. (6pts)  $\frac{d}{dx} (x^2 + 3) \ln(x^2 + 3) =$

7. (5pts) Let  $f(x) = \cos(3x)$ . What is  $f^{(71)}(x)$ , the 71st derivative of  $f$ ? Justify your answer.

Use L'Hopital's rule to find the following limits:

8. (6pts)  $\lim_{x \rightarrow \infty} \frac{x^3}{e^x} =$

9. (10pts)  $\lim_{x \rightarrow \infty} (x^2 + x - 2)^{\frac{1}{x}} =$

Find the following antiderivatives.

10. (7pts)  $\int 3x^8 - \frac{1}{1+x^2} + \sqrt[4]{x^{17}} + \pi^4 dx =$

11. (3pts)  $\int e^{5x-7} dx =$

12. (7pts)  $\int \frac{x^2 + 1}{\sqrt{x}} dx =$

Use the substitution rule in the following integrals:

13. (7pts)  $\int \frac{2x - 3}{x^2 - 3x + 1} dx =$

14. (10pts)  $\int_0^{\frac{\pi}{6}} \frac{\sin x}{\cos^3 x} dx =$

**15.** (8pts) Find the equation of the tangent line to the curve  $y = x^2 + 3x - 10$  at the point  $(1, -6)$ . Sketch the curve and the tangent line on the same graph.

**Bonus.** (10pts) The rear inside cover of our book claims that

$$\int \frac{x^2 dx}{\sqrt{a^2 - x^2}} = -\frac{x}{2}\sqrt{a^2 - x^2} + \frac{a^2}{2} \arcsin \frac{x}{a} + C$$

Verify this formula. *Hint: it's not about figuring out the way to do the integral.*