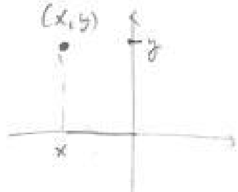
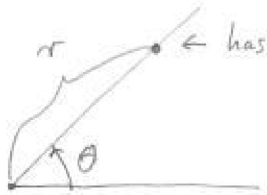
10.4 Polar coordinates

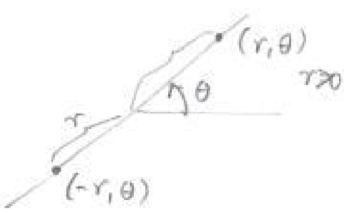
We describe the position of a pt in the plane with (x,y) coordinates



Another way

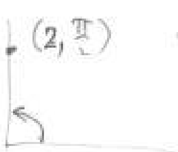


We extend polar coord. to negative r



so: $(-r,\theta)$, $(r,\theta+\pi)$ represent same point.

Note: A point may have many different polar coordinates



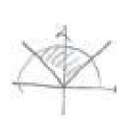
other coord: $(2,-\frac{3\pi}{2})$

$(-2,\frac{3\pi}{2})$

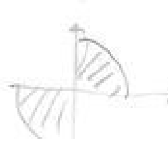
$(-2,-\frac{\pi}{2})$ etc.

Ex: Plot points $(1,\frac{3\pi}{4})$, $(-1,\frac{7\pi}{6})$, $(3,-\frac{2\pi}{3})$, $(-2,-\frac{4\pi}{3})$

Ex: Plot regions: a) $0\le r\le 1$, $\frac{\pi}{4}\le\theta\le\frac{3\pi}{4}$

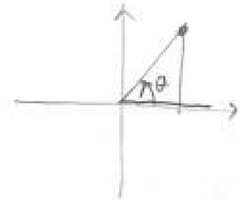


b) $-1\le r\le 1$ $\quad 0\le\theta\le\frac{\pi}{2}$



Conversion of coordinates:

assume pole is placed at the origin
x-axis = polar axis



polar → cart.

$x = r\cos\theta$

$y = r\sin\theta$

cart → polar

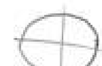
$\tan\theta = \frac{y}{x}$ ← choose θ so (r,θ) is in correct quadrant.

$r=\sqrt{x^2+y^2}$

Ex: $(2,\frac{5\pi}{4}) \to$ cart $\qquad (-1,1)\to$ polar

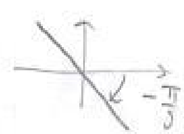
Polar curves

Ex: Sketch: a) $r=3$



circle

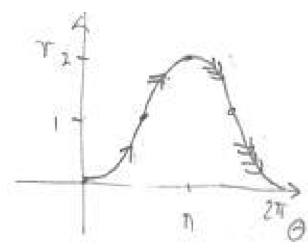
b) $\theta=-\pi/3$

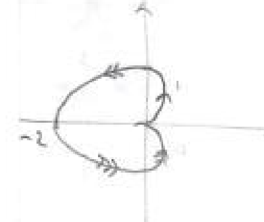


line

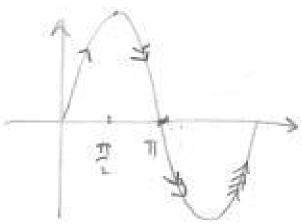
Typically, curves are given by $r=f(\theta)$

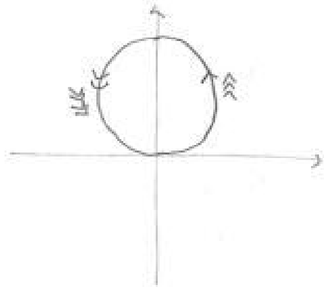
Ex: $r=1-\cos\theta$





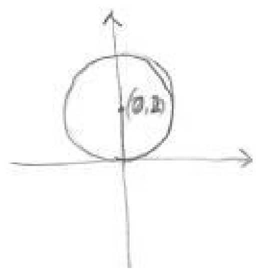
Ex: $r = 4\sin\theta$



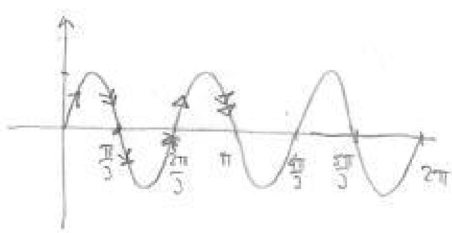


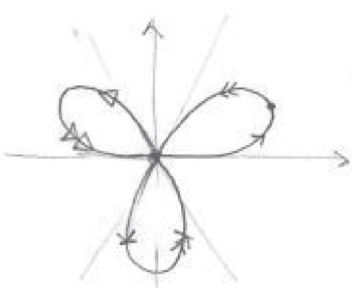
We can verify it is a circle using coord.

$$r = 4\sin\theta \quad /\cdot r$$
$$r^2 = 4r\sin\theta$$
$$x^2 + y^2 = 4y$$
$$x^2 + y^2 - 4y = 0$$
$$x^2 + (y-2)^2 = 4$$



Ex: $r = \sin 3\theta$





In general: $r = a\sin n\theta$, $a\cos n\theta$ is a rose with $\begin{cases} n \text{ petals if } n \text{ is odd} \\ 2n \text{ petals if } n \text{ is even} \end{cases}$

(traced out once over $[0, 2\pi]$)