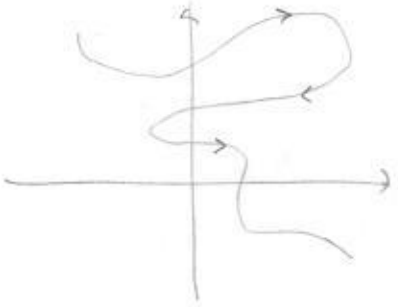
10.1 Curves defined by parametric equations

Problem: equations such as $y = f(x)$ cannot describe the curve below, since the curve doesn't pass the vertical line test.



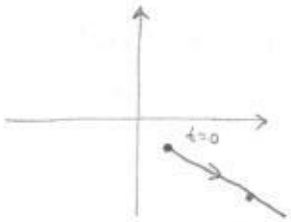
What we can do: view path as motion of a particle. We will know the position if we know the x and y coordinates at every moment, that is if we are given

$$x = f(t), \quad y = g(t), \quad a \le t \le b$$

parametric equations of the curve

t is called the parameter - usually thought of as time.

Ex: $x = 1 + 3t$, $t \ge 0$
$y = -1 - 2t$



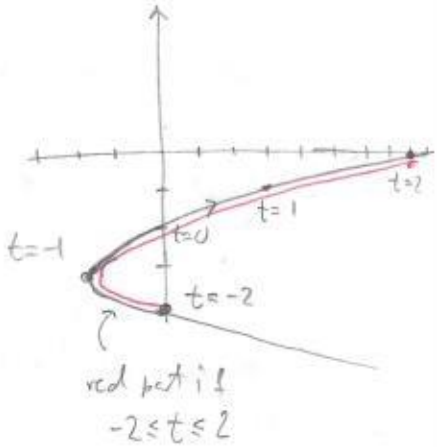
Eliminate parameter:

$$y = -1 - 2\frac{x-1}{3} = -1 - \frac{2}{3}(x-1) = -\frac{2}{3}x - \frac{1}{3}$$

Note: The parametric curve is only a portion of the curve you get once the parameter is eliminated. Extra effort is needed to see which portion.

Ex: $y = t - 2$
$x = t^2 + 2t$

t	x	y
-2	0	-4
-1	-2	-3
0	0	-2
1	3	-1
2	8	0

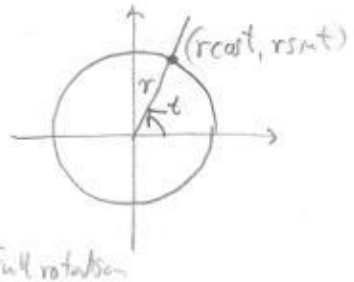


red part is $-2 \le t \le 2$

$$x = (y+2)^2 + 2(y+2)$$
$$= y^2 + 2y + 4 + 2y + 4 = y^2 + 4y + 8$$

Note: "t" is not visible in the picture, (not on any coordinate axis)

Ex: $x = r\cos t$
$y = r\sin t$



$x^2 + y^2 = r^2$

$0 \le t \le 2\pi$ is one full rotation

How to get clockwise direction?

$x = r\cos t$
$y = -r\sin t$

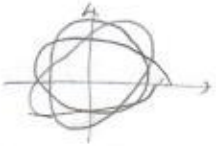
Or make it start at $(0,1)$?

$x = r\sin t$
$y = r\cos t$

Ex: Use calculator to graph:

a) $x = 4\cos t + \cos 4t$
$y = 4\sin t - \sin 4t$

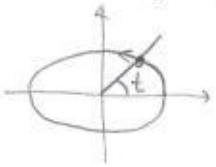
b) $x = 4\cos 3t + \cos 4t$
$y = 4\sin 3t - \sin 4t$



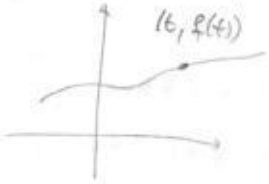
c) $x = 4\cos 3t + \cos 4t$
(opt.) $y = 4\sin t - \sin 4t$

Ex: An ellipse is parametrized by

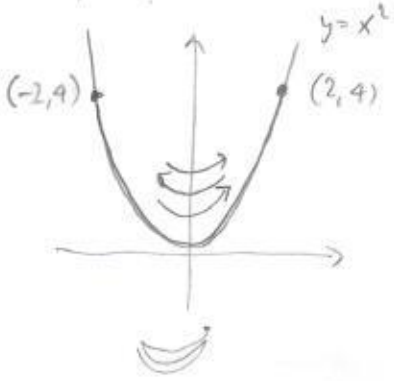
$x = a\cos t$
$y = b\sin t$



Note: Graph of the function $y = f(t)$ parametrized by
$x = t$
$y = f(t)$

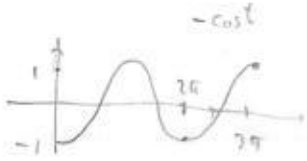


Ex: Write parametric equations that describe the motion:

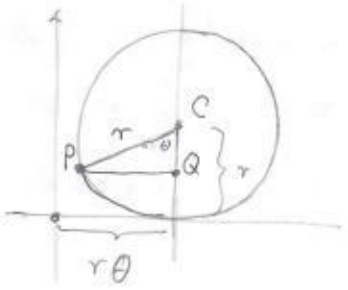


$x = -2\cos t$
$y = 4\cos^2 t$

$0 \le t \le 3\pi$



Ex: The curve traced out by a circle as it rolls along a straight line is called a cycloid. Parametrize it.



$|PQ| = r\sin\theta$
$|CQ| = r\cos\theta$

$x = r\theta - |PQ| = r(\theta - \sin\theta)$
$y = r - |CQ| = r(1 - \cos\theta)$