1. (14pts) Let A, B and C be subsets of some universal set U.
a) Use Venn diagrams to draw the following subsets (shade).
b) Among the four sets, two are equal. Use set algebra to show they are equal.

$$(B \cap C) - A \qquad (A \cap B) \cap C \qquad (B - A) \cap C \qquad (A - B) \cup (B - C)$$

2. (12pts) Let U be the set of integers. Consider the sets $A = \{k \in \mathbf{Z} \mid k \equiv 2 \ (\mathrm{mod}\ 4)\}$, $B = \{k \in \mathbf{Z} \mid k \text{ is divisible by } 4\}$, $C = \{k \in \mathbf{Z} \mid k < 0\}$ and write the following subsets using the roster method (pattern needs to be obvious).

$A \cap C =$

$B - C =$

$C^c =$

$(A \cup B) \cap C =$

$C - (A \cup B) =$

$B - A =$

3. (12pts) Let $A = \{k \in \mathbf{Z} \mid k \equiv 1 \pmod 3\}$ and $B = \{k \in \mathbf{Z} \mid k \equiv 4 \pmod 6\}$.
a) Is $A \subseteq B$? Prove or disprove.
b) Is $B \subseteq A$? Prove or disprove.

4. (16pts) Let $f : \mathbf{R} \times \mathbf{R} \to [0, \infty)$ be given by $f(x, y) = x^2 + y^2$.
a) Is f surjective? Justify.
b) Is f injective? Justify.
c) Determine the set of preimages of 5. List at least three elements of this set and illustrate it in the plane.

5. (14pts) Let $\mathbf{Z}_4 = \{0, 1, 2, 3\}$, and let $f, g : \mathbf{Z}_4 \to \mathbf{Z}_4$, $f(x) = x^2 + 4x \pmod 4$, $g(x) = x^2 - 4$ $\pmod 4$.

a) Write the table of function values for f and g.

b) The formulas for f and g are different. Are the functions f and g equal?

c) What is the set of preimages of 3 under f?

d) What is the set of preimages of 0 under f?

e) Show that $x^2 + 4x \equiv x^2 - 4 \pmod 4$ for every $x \in \mathbf{Z}_4$. This implies that $f(x) = g(x)$ for every $x \in \mathbf{Z}_4$.

6. (10pts) Let $f(x) = (x - 2)^2 + 7$ and assume the codomain is $\mathbf{R}$.

a) What subset of real numbers is the natural domain for this function?

b) What is the range of this function? Justify your answer.

7. (10pts) Draw arrow diagrams between two copies of **Z** below that illustrate a function $f : \mathbf{Z} \to \mathbf{Z}$ that is:

a) a surjection that is not an injection

$\ldots -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \ldots$

b) an injection that is not a surjection

$\ldots -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \ldots$

$\ldots -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \ldots$

$\ldots -3 \quad -2 \quad -1 \quad 0 \quad 1 \quad 2 \quad 3 \ldots$

8. (12pts) Let A, B be subsets of a universal set U. Prove that $A \subseteq B$ if and only if $A \cup B = B$.

Bonus. (10pts) Let $A = \{x \in \mathbf{R} \mid x \neq -1, 1\}$ and let $f : A \to \mathbf{R}$, $f(x) = \dfrac{2}{1 - x^2}$. Determine the range of f.