1. (14pts) Prove: the sum of squares of three consecutive integers always gives remainder 2 when divided by 3.

2. (14pts) Prove using induction: for every integer $n \geq 0$, $1 + 3 + 3^2 + \cdots + 3^n = \dfrac{3^{n+1} - 2}{2}$.

3. (16pts) Let $a, b \neq 0$ be real numbers. Two of the following statements are true, and one is false. Prove the true ones (one is basic and needs only a little explanation), and justify why the remaining one is false.

a) If a and b are rational, then ab is rational.

b) If a is rational and b is irrational, then ab is irrational.

c) If a and b are irrational, then ab is irrational.

4. (18pts) Consider the statement: for every integer n, n is divisible by 8 if and only if n^2 is divisible by 8.

a) Write the statement as a conjunction of two conditional statements.

b) Determine whether each of the conditional statements is true, and write a proof, if so.

c) Is the original statement true?

5. (14pts) We have shown on homework: for every integer n, if n^2 is even, then n is even. Use this proposition to show directly that $\sqrt{8}$ is irrational, that is, **without** using the fact that $\sqrt{2}$ is irrational.

6. (10pts) Use the triangle inequality to prove that for all real numbers c, d,

$2|c| \leq |c + d| + |c - d|.$

7. (14pts) Prove that for all real numbers x, y, $x^2 + y^2 \geq 6x - 9$.

Bonus. (10pts) Show that the number 345,237,211,897,873,929,146 is not a square of any integer. *Hint: use congruence, but* (mod 5) *and* (mod 10), *like in our homework problem, will not work.*