Mathematical Reasoning — Exam 1
MAT 312, Fall 2017 — D. Ivanšić

Name: _____

Show all your work!

Consider the following sentences. If a sentence is a statement, determine whether it is true (and justify your answer). If it is an open sentence, find its truth set.

1. (2pts) $2 < 5$ or $3 \geq 5$

2. (2pts) If the square of a real number is negative, then that number is negative.

3. (3pts) (universal set=$\mathbf{Z}$) $2x^2 = 5x + 3$

4. (4pts) There exists a $y \in \mathbf{R}$ such that $y^3 + y - 5 = 0$.

5. (3pts) (universal set=$\mathbf{R}$) $3x + 5 < 2x + 4$

Negate the following statements.

6. (3pts) If you know a little bit of English, you can go far.

7. (3pts) Kim Jong Un launches a rocket and takes a bath.

8. (8pts) Use a truth table to prove that $(\neg P \vee Q) \wedge (P \vee \neg Q) \wedge \neg P \equiv \neg P \wedge \neg Q$. (Use however many columns you need.)

P	Q								
T	T								
T	F								
F	T								
F	F								

9. (12pts) Use previously proven logical equivalences to prove the equivalence $(P \Longrightarrow Q) \Longrightarrow R \equiv (\neg P \Longrightarrow R) \wedge (Q \Longrightarrow R)$. Do not use a truth table.

10. (4pts) Write the converse and contrapositive of the statement: if a real number is greater than 5, then its absolute value is greater than 5.

Converse:

Contrapositive:

11. (8pts) Suppose the following statements are true:
If I ate ice cream, then I ate strawberries.
I did not eat ice cream or I did not eat strawberries.

Determine truth value of the following statement and justify: I ate ice cream.

12. (4pts) Use the roster method to write the set $\{x \in \mathbf{Z} \mid x^2 \text{ is odd and smaller than } 30\}$.

13. (7pts) An integer n is divisible by 7 if $n = 7k$ for some integer k.
a) Write the definition using symbols for quantifiers.
b) Negate the definition using symbols for quantifiers.
c) Finish the sentence: "An integer n is not divisible by 7 if ..."

14. (10pts) There exists an integer n such that for every integer m, if $mn = 24$, then $|n| < 6$.
a) Write this statement using symbols.
b) Write the negation of the statement using symbols.
c) Write the negation of the statement in English.

15. (12pts) Let $\mathbf{Z}$ be the universal set. The following is an open sentence in x:

$$(\exists y \in \mathbf{Z})(x + 2y = 5)$$

a) If $x = 1$, is the statement true?
b) If $x = 6$, is the statement true?
c) Find the truth set (the x's) of the above statement.

16. (15pts) We will call an integer n type-0, type-1, type-2, or type-3 if it can be written in the form $n = 4k$, $n = 4k + 1$, $n = 4k + 2$, or $n = 4k + 3$, respectively, for some integer k. Show that if n is a type-3 integer, then $n^2 - n$ is a type-2 integer. Start with a know-show table if you find it helpful.

Bonus. (10pts) Consider the general quadratic equation $ax^2 + bx + c = 0$. Prove the following statement: if $a > 0$, $b < 0$ and $c > 0$ and the equation has a real solution, then both solutions are positive.