Calculus 2 — Final Exam
MAT 308, Spring 2020 — D. Ivanšić

Name: _____

Show all your work!

If you are filming yourself as you take the exam for later upload, write **code: 32FE5Y** on the first sheet of paper with your solutions. Then hold the paper at the beginning so the code can be captured by the camera.

Find the following integrals:

1. (6pts) $\int xe^{2x}\,dx =$

2. (10pts) $\int \sec^4 x \tan^3 x\,dx =$

3. (12pts) Use trigonometric substitution to evaluate the integral.

$$\int \frac{x^3}{\sqrt{9-x^2}}\,dx =$$

4. (6pts) Determine whether the following improper integral converges, and, if so, evaluate it.

$$\int_0^\infty \frac{1}{1+x^2}\,dx =$$

5. (16pts) The region bounded by the curves $y = x^2 - 4x$ and $y = 30 - x^2$ is rotated around the x-axis.

a) Sketch the solid and a typical cross-sectional washer.

b) Set up the integral for the volume of the solid. Simplify, but do not evaluate the integral.

6. (10pts) Justify why the series converges and find its sum.

$$\sum_{n=0}^{\infty} \frac{2^{2n+1}}{7^{n-1}} =$$

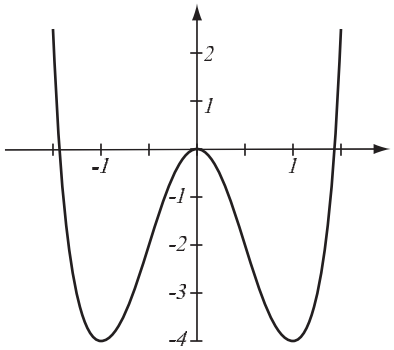
7. (14pts) Find the interval of convergence of the series. Don't forget to check the endpoints.

$$\sum_{n=0}^{\infty} \frac{(x-3)^n}{2^{n+1}(n+4)}.$$

8. (16pts) Let $f(x) = \ln x$.
a) Find the 3rd Taylor polynomial for f centered at $a = 10$.
b) Use Taylor's formula to get an estimate of the error $|R_3|$ on the interval $(8, 12)$.

9. (10pts) A particle moves along the path with parametric equations $x(t) = \cos t$, $y(t) = 4 + \sin^2 t$, $0 \leq t \leq 2\pi$. Eliminate the parameter in order to sketch the path of motion and then describe the motion of the particle.

10. (24pts) The integral $\int_0^1 \cos(x^2)\,dx$ is given. It cannot be found by antidifferentiation, since the antiderivative of $\cos(x^2)$ is not expressible using elementary functions.
a) Write the expression you would use to calculate M_6, the midpoint rule with 6 subintervals. All the terms need to be explicitly written, do not use f in the sum.
b) The graph of y'' is shown: use it to find the error estimate for M_n in general.
c) What should n be in order for M_n to give you an error less than 10^{-4}?
d) Use a known power series for to find a power series for the above integral.
e) How many terms of the power series are needed to estimate the integral to accuracy 10^{-4}? Write the estimate as a sum (you do not have to simplify it).
f) Which method requires less computation to evaluate the integral with accuracy 10^{-4}, midpoint formula or series?



11. (10pts) In another attempt to fight the coronavirus, a bottle of disinfectant is thrown from the origin so that its position is given by $x(t) = 15t$, $y(t) = 16t - 5t^2$, where length is measured in meters, time in seconds. Find the equation of the tangent line to this curve when $t = 2$.

12. (16pts) Find the area inside the polar curve $r = 2\cos\theta$ and outside $r = 1$. Draw a picture showing the area you are computing.

Bonus. (15pts) The graph of the parametric curve $x(t) = t^3 - 12t$, $y(t) = -t^2 - 2t + 8$ is shown. Compute the area enclosed by the loop.