Calculus 2 — Exam 3
MAT 308, Spring 2020 — D. Ivanšić

Name: _____

Show all your work!

Find the limits, if they exist.

1. (6pts) $\lim_{n\to\infty} \frac{4^{n-1}}{3^{2n}} =$

2. (6pts) $\lim_{n\to\infty} \cos\frac{n\pi}{2} =$

3. (10pts) Find the limit. Use the theorem that rhymes with what a person might do, if an irritant enters their nose.

$$\lim_{n\to\infty} \frac{\sin n + 2\cos n}{3n - 7}$$

4. (6pts) Write the series using summation notation:

$$\frac{9}{1} - \frac{27}{1 \cdot 2} + \frac{81}{1 \cdot 2 \cdot 3 \cdot 4} - \frac{243}{1 \cdot 2 \cdot 3 \cdot 4 \cdot 5 \cdot 6} + \cdots =$$

5. (12pts) Justify why the series converges and find its sum.

$$\sum_{n=2}^{\infty} (-1)^n \frac{5^{n-1}}{3^{2n+1}} =$$

Determine whether the following series converge and justify your answer.

6. (6pts) $\displaystyle\sum_{n=1}^{\infty} e^{\frac{1}{n}}$

7. (12pts) $\displaystyle\sum_{n=1}^{\infty} \frac{\sqrt{n}+3}{n^2-2n-3}$

8. (22pts) Consider the alternating series $\sum_{n=2}^{\infty}(-1)^n \frac{n^2}{n^3+7}$.

a) Show that the sequence $\frac{n^2}{n^3+7}$ is decreasing from some point on.

b) Show the limit of the sequence in a) is 0.

c) Is the series convergent?

d) Is the series absolutely convergent? Use the integral test.

Determine whether the following series converge using the root or ratio test.

9. (10pts) $\sum_{n=3}^{\infty} \frac{5^{3n}}{(n+1)!}$

10. (10pts) $\displaystyle\sum_{n=1}^{\infty}(-1)^n\frac{(\arctan n)^n}{n^2+4n}$

Bonus. (10pts) Does $\displaystyle\sum_{n=0}^{\infty}\frac{2^n+3^n}{4^n+5^n}$ converge? (Hint: root test and dominant terms.)